Markov availability — a repaired machine's availability, with constant or wear-out failure rates

Menu: System ▸ Markov Availability...

Part 1 — What the function does

What it calculates. A Markov model describes a repaired machine as a set of states - up, down, worn, under repair - and the rates at which it moves between them. ReliaEn solves the model and answers, for the machine you describe:

Result	How ReliaEn sets it
Steady-state availability A(∞)	The fraction of the time the machine is up in the long run
Mission availability A(t)	The chance that it is up at the mission time t
Mission reliability R(t)	The chance that it has never been down by t
MTTF (first entry into Down)	The mean time to the first failure
Expected downtime / yr at A(∞)	(1 − A(∞)) × the operating hours per year

How it solves the model depends on the failure rate:

Model source	Failure rate	How ReliaEn calculates
Preset machine or Custom (worksheet)	Constant (λ per hour)	The exact state probabilities P(t) = P(0)·e^(Qt) and the steady state πQ = 0
Wear-out (Weibull β, η)	Growing with age	Four methods side by side, one for each thing a repair can do to the machine: Method 1 minimal repair ("as bad as old", exact), Method 2 a chain of wear stages (renewal, approximate in the transient), Method 3 semi-Markov (renewal, exact), Method 4 simulation with a virtual age (any repair from as good as new, q = 0, to as bad as old, q = 1)

With a target availability the report adds how fast repairs must be, or how many units are needed, to reach it; with a production loss per hour it prices the downtime. A seeded simulation of the same model checks the answer.

From which data. For a preset or wear-out machine, every value is typed into the form: the rates or the Weibull shape β and scale η, the mean time to repair (MTTR), the mission time and the operating hours per year. For a custom chain, one worksheet row per transition: the state it leaves, the state it enters, and the rate. β, η and the MTTR can also come from fits made earlier in the session.

When an engineer uses it. When a repaired machine's availability is the question - a downtime budget, a target to meet, or how good the repairs must be on a machine that wears out.

When not to use it.

Situation	Use instead
A system that is not repaired, its reliability from a block diagram	System ▸ RBD Canvas (chapter 4.1)
A large system with repair crews, spares and preventive maintenance	System ▸ RAM Simulation... (chapter 4.5)

β and η not yet known	Fit them first with Life Data ▸ Time Dependence ▸ Weibull (2P) distribution..., or from a repair history with Life Data ▸ Repairable Systems ▸ GRP / Kijima...; then pick the fit in λ source

---

Part 2 — The example's input data

Setting. A conveyor gearbox wears out: its life follows a Weibull distribution with shape 2 and scale 1,000 hours. It is repaired every time it fails, in 50 hours on average. The maintenance engineer wants its availability over 4,000 hours of running - and has to say honestly what a repair does to the gearbox.

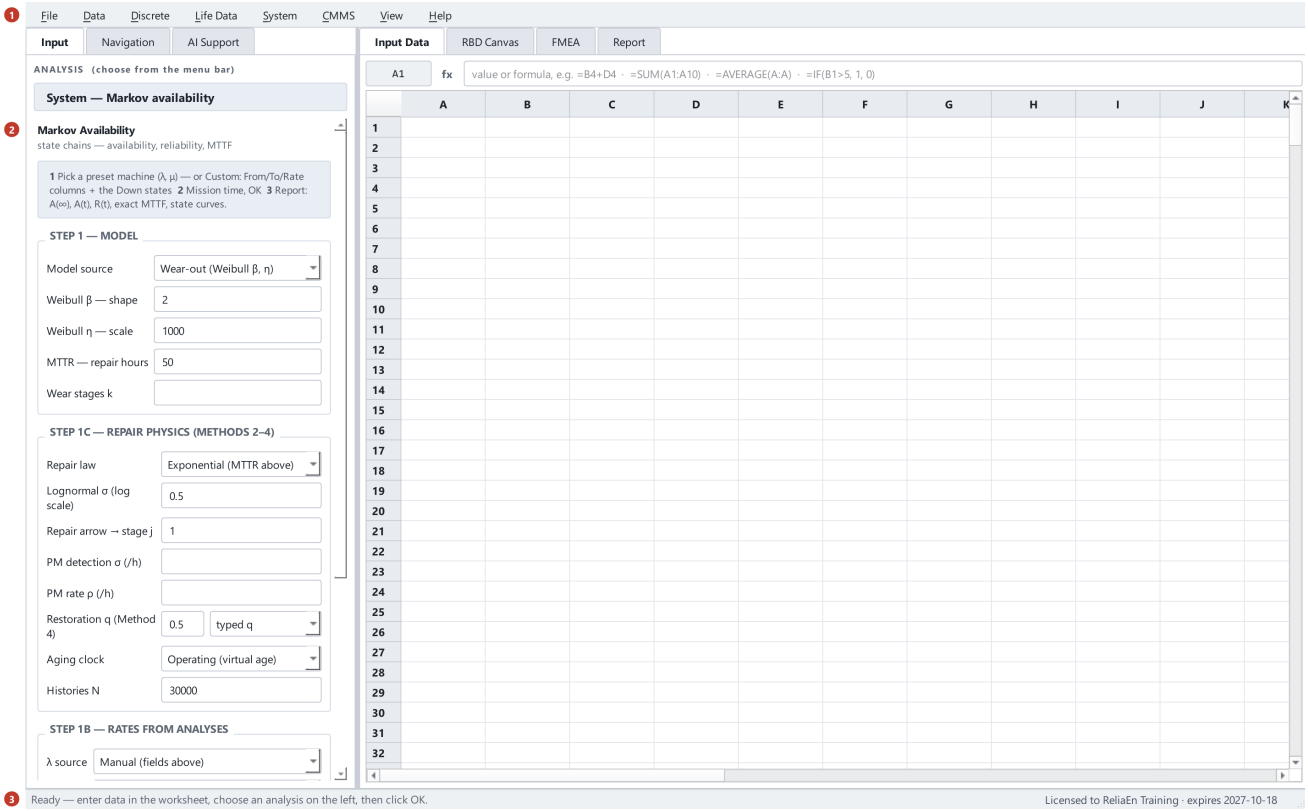
Input	Value
Model source	Wear-out (Weibull β, η)
Weibull β — shape, Weibull η — scale	2, 1000
MTTR — repair hours	50
Wear stages k	blank (ReliaEn chooses)
Repair law	Exponential (MTTR above)
Repair arrow → stage j	1
Restoration q (Method 4)	0.5 (typed q)
Aging clock	Operating (virtual age)
Histories N	30000
Mission time t	4000
Operating hours / yr	8760

Second example, constant rates. A single process pump fails at 0.002 per hour and is repaired at 0.05 per hour (a 20-hour mean repair). The plant wants 99% availability, and one hour of downtime costs 10,000 baht.

Input	Value
Model source, Preset	Preset machine, 1 unit repairable
λ — failure rate, μ — repair rate	0.002, 0.05
Mission time t, Operating hours / yr	10000, 8760
Production loss / hr	10000
Target availability %	99

No worksheet and no file: both examples are typed into the form as Part 3 shows.

The gearbox's form set as Part 3's steps 2 to 17 set it, in the window (the pump's form is the picture of steps 19 to 29):



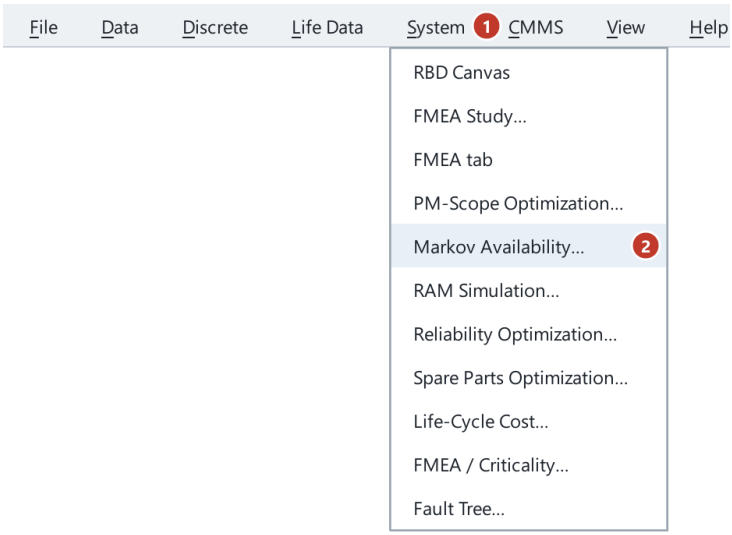
The gearbox's values in the window

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set with the values of the table above; OK — Calculate at its foot.
- ③ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 3 — How to use it, step by step

The gearbox (wear-out).

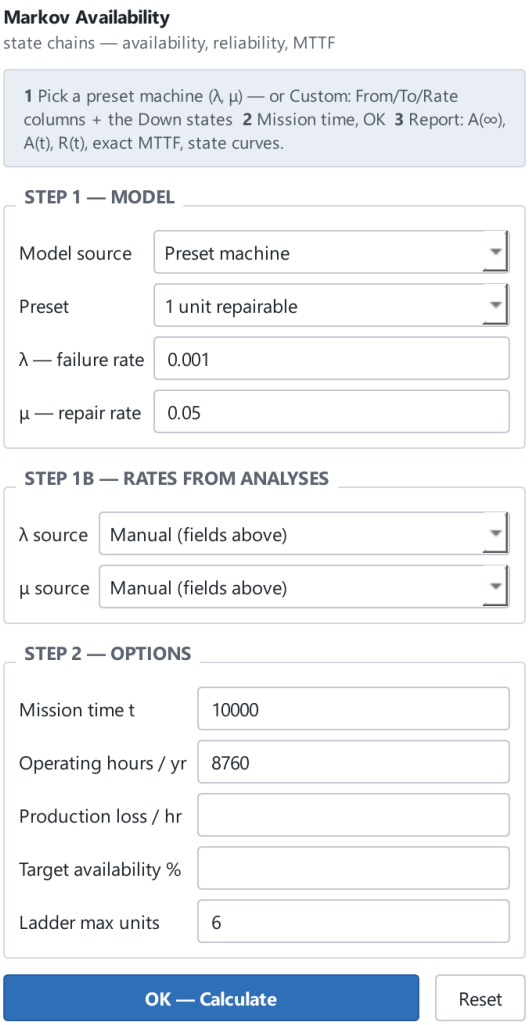
1. **Choose System ▸ Markov Availability…** in the menu bar. The form Markov Availability opens in the left panel.



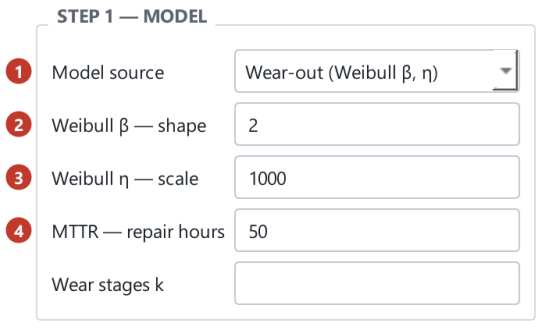
Step 1 — the menu path

- ① System: the menu.
- ② Markov Availability…: the entry to click.

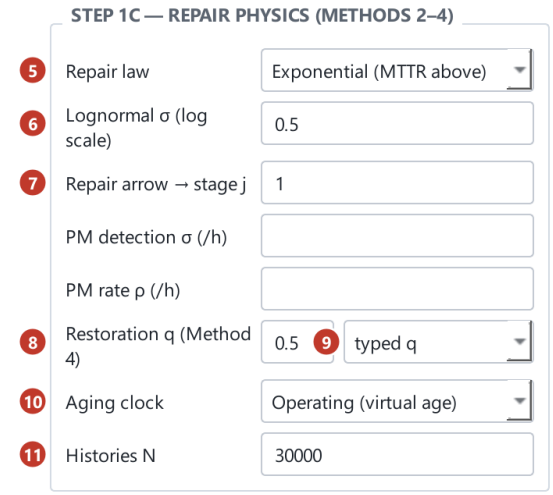
The form as it opens, before any box is set:



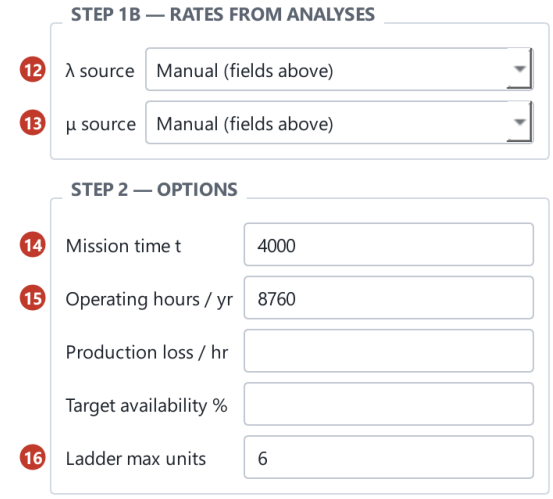
Step 1 — the form as it opens



Steps 2 to 5 — the gearbox's model boxes, ① to ④



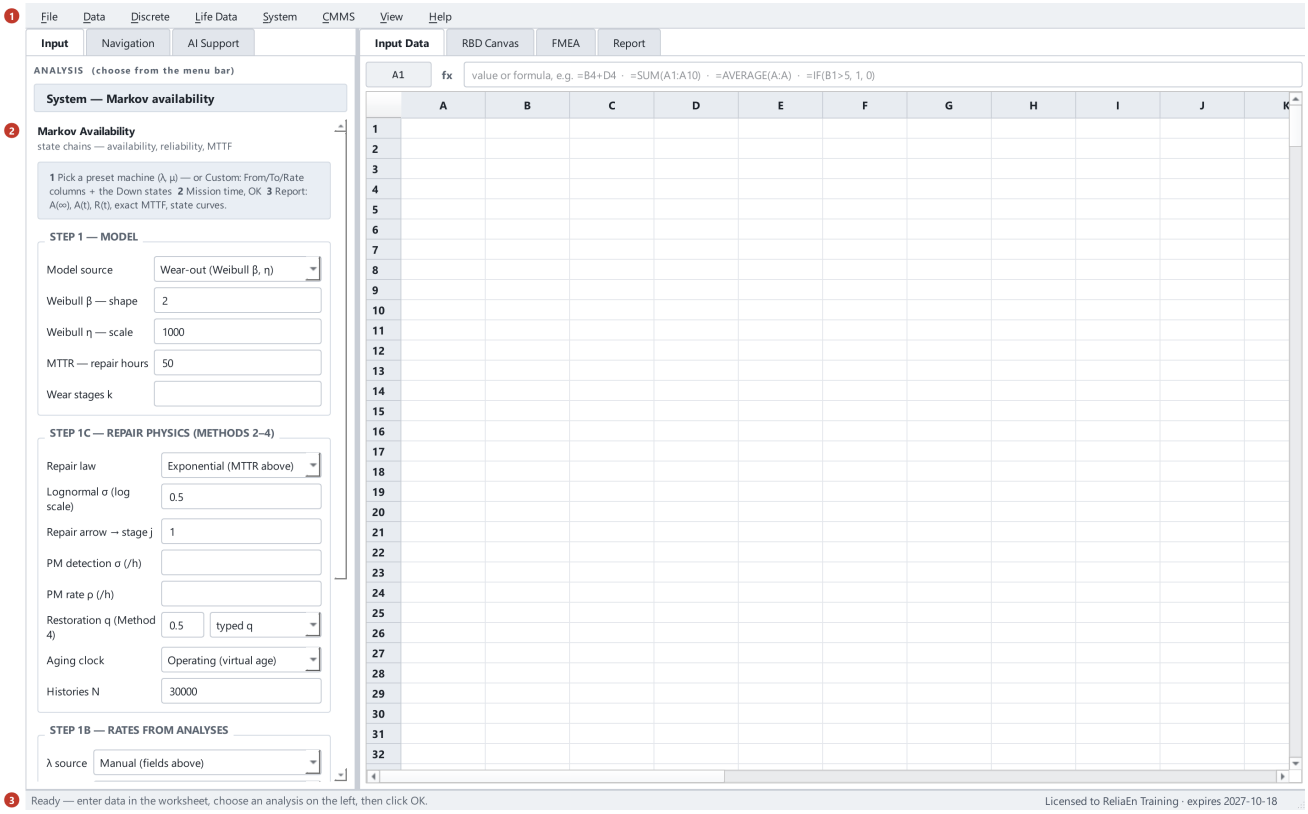
Steps 6 to 12 — its repair boxes, ⑤ to ⑪



Steps 13 to 17 — its rate sources and options, ⑫ to ⑯

2. ① **In Model source, choose Wear-out (Weibull β, η).** The wear-out boxes appear.
3. ② **In Weibull β — shape, type 2.**
4. ③ **In Weibull η — scale, type 1000.**
5. ④ **In MTTR — repair hours, type 50.** Wear stages k stays blank: ReliaEn chooses k.
6. ⑤ **In Repair law, keep Exponential (MTTR above).**
7. ⑥ **Lognormal σ (log scale) keeps 0.5;** it is used only with a lognormal repair law.
8. ⑦ **In Repair arrow → stage j, keep 1:** a repair returns the gearbox to the first stage, as new.
9. ⑧ **In Restoration q (Method 4), type 0.5.**
10. ⑨ **Keep typed q** as the source of q.
11. ⑩ **In Aging clock, keep Operating (virtual age).**
12. ⑪ **In Histories N, type 30000.**
13. ⑫ **In λ source, keep Manual (fields above).**
14. ⑬ **In μ source, keep Manual (fields above).**
15. ⑭ **In Mission time t, type 4000.**
16. ⑮ **In Operating hours / yr, keep 8760.**
17. ⑯ **Ladder max units keeps 6;** it is used only with a target. Production loss / hr and Target availability % stay blank for the gearbox.

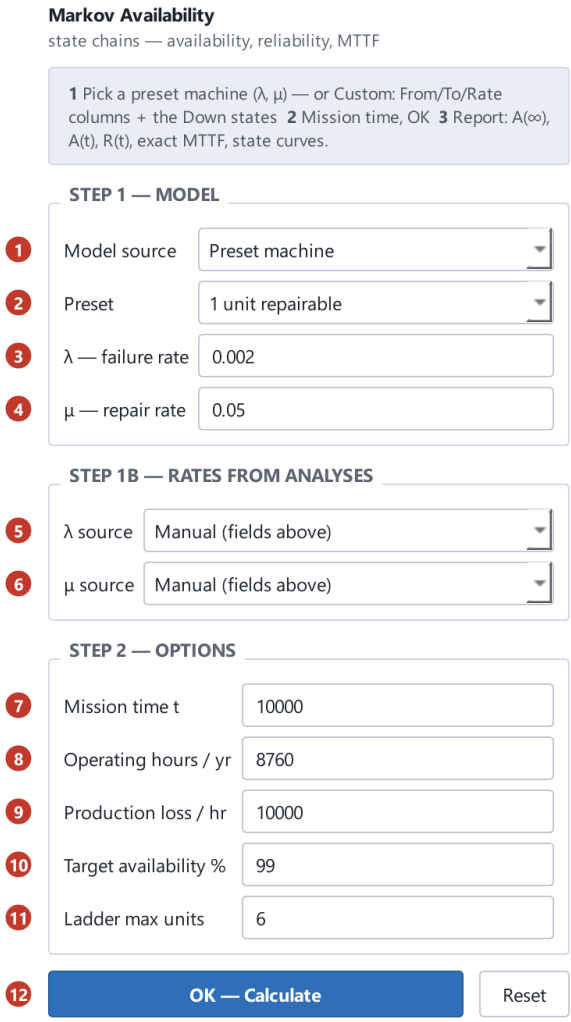
18. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates the four methods and shows the report (Part 4, first half).



Step 18 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 2 to 17; OK — Calculate at its foot.
- ③ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

The pump (constant rates). The same form; steps 19 to 29 set its boxes, marked ① to ⑪; ⑫ is the button of step 30:



Steps 19 to 29 — the form set for the pump

19. ① **In Model source, choose Preset machine.**
20. ② **In Preset, choose 1 unit repairable.**
21. ③ **In λ — failure rate, type 0.002.**
22. ④ **In μ — repair rate, type 0.05.**
23. ⑤ **In λ source, keep Manual (fields above).**
24. ⑥ **In μ source, keep Manual (fields above).**
25. ⑦ **In Mission time t, type 10000.**
26. ⑧ **In Operating hours / yr, keep 8760.**
27. ⑨ **In Production loss / hr, type 10000.**
28. ⑩ **In Target availability %, type 99.**
29. ⑪ **In Ladder max units, keep 6.**
30. ⑫ **Click OK — Calculate.** The report of Part 4's second half.

Part 4 — The report, card by card

The gearbox (wear-out), after step 18:

	INPUT DATA	
1	Failure model	`Weibull (β = 2, η = 1000) — hazard λ(t) = (β/η)(t/η)^(β−1)`
2	Weibull β / η	`2 / 1000`
3	Repair	`manual MTTR — MTTR = 50 h (μ = 0.02/h)`
4	Mission time t	`4000`
5	Wear stages k	`4 (auto from CV² = 0.2732)`
6	Repair law (Methods 3–4)	`exponential`
7	Repair arrow (Method 2)	`Stage 1`
8	Method 4	`q = 0.5, operating aging, N = 30,000 histories, seed 20260825`

Input data

- ① Failure model Weibull (β = 2, η = 1000) — hazard λ(t) = (β/η)(t/η)^(β−1). The gearbox wears out: its hazard grows with age.
- ② Weibull β / η 2 / 1000.
- ③ Repair manual MTTR — MTTR = 50 h (μ = 0.02/h). The mean time to repair (MTTR) and its rate.
- ④ Mission time t 4000.
- ⑤ Wear stages k 4 (auto from CV^2 = 0.2732). ReliaEn chose four stages to match the spread of the Weibull life.
- ⑥ Repair law (Methods 3–4) exponential. Repair arrow (Method 2) Stage 1: a repair returns the machine to the first stage, as new.
- ⑦ Repair arrow (Method 2) Stage 1.
- ⑧ Method 4 q = 0.5, operating aging, N = 30,000 histories, seed 20260825.

	RESULTS	
1	MTTF = η·Γ(1+1/β)	`886.23 h`
2	A(t) — Method 1, MINIMAL repair	`0.7161`
3	A(t) — Method 2, RENEWAL repair	`0.9466`
4	A(∞) — renewal identity MTTF/(MTTF+MTTR)	`0.94659`
5	A(∞) — minimal repair	`none — availability erodes forever`
6	A(t) — Method 3, RENEWAL repair (semi-Markov, exact)	`0.9466`
7	A(t) — Method 4, restoration q = 0.5 (operating aging)	`0.8383 ± 0.0042`

Results

- ① MTTF = η·Γ(1+1/β) 886.23 h. The mean life of a new gearbox.
- ② A(t) — Method 1, MINIMAL repair 0.7161. The availability at 4,000 hours if each repair only patches the gearbox.
- ③ A(t) — Method 2, RENEWAL repair 0.9466. The availability if each repair renews it.
- ④ A(∞) — renewal identity MTTF/(MTTF+MTTR) 0.94659. A(∞) — minimal repair none — availability erodes forever.
- ⑤ A(∞) — minimal repair none — availability erodes forever.
- ⑥ A(t) — Method 3, RENEWAL repair (semi-Markov, exact) 0.9466.
- ⑦ A(t) — Method 4, restoration q = 0.5 (operating aging) 0.8383 ± 0.0042. A repair that removes half of the age gained since the last one.

INTERPRETATION

1 Same machine, two repair philosophies, two answers: with MINIMAL repair (fix the part, keep the age) availability at t = 4000 is 0.7161 and still falling — maintenance is a losing race against the growing hazard. With RENEWAL repair (replace, as good as new) it settles at 0.9466. Neither is 'more correct' — state honestly what your repairs do to the machine, and read that column. Method 3 gives the renewal transient exactly (the same steady state, reached through the failure waves Method 2 blurs); Method 4's restoration dial q spans the two physics — everything between them is imperfect repair, reachable only by simulation.

Interpretation

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WHICH REPAIR PHYSICS? — FOUR METHODS, ONE MACHINE

1 Same machine, four answers — each answers a different physical question; the most consequential modelling decision here is not mathematical.

2 Method	3 Where the memory lives	4 Repair physics encoded	5 Answer quality	6 This machine's long-run availability
7 1. Non-homogeneous Markov	Global clock time in the rates, Q(t)	MINIMAL repair (as bad as old)	Exact for its philosophy	0.5006 at 10,000 h
8 2. Method of stages	Which sub-state you occupy	Renewal, or partial (arrow choice)	Approximate (CV² matching)	0.94659
9 3. Semi-Markov	A local clock that resets at each jump	RENEWAL (as good as new), built in	Exact for its philosophy	0.94659
10 4. Simulation, virtual age	A per-component virtual age V	ANY: q dial spans minimal ↔ renewal	Statistical (with CIs)	0.838 at the chosen q = 0.5 (4,000 h)

11 Choosing the method is choosing the repair physics — Method 1: minimal repair; Methods 2 and 3: renewal; Method 4: any q.

Which repair physics? — four methods, one machine

- ① Same machine, four answers — each answers a different physical question; the most consequential modelling decision here is not mathematical.
- ② Method
- ③ Where the memory lives
- ④ Repair physics encoded
- ⑤ Answer quality
- ⑥ This machine's long-run availability
- ⑦ 1. Non-homogeneous Markov · Global clock time in the rates, Q(t) · MINIMAL repair (as bad as old) · Exact for its philosophy · 0.5006 at 10,000 h
- ⑧ 2. Method of stages · Which sub-state you occupy · Renewal, or partial (arrow choice) · Approximate (CV² matching) · 0.94659
- ⑨ 3. Semi-Markov · A local clock that resets at each jump · RENEWAL (as good as new), built in · Exact for its philosophy · 0.94659
- ⑩ 4. Simulation, virtual age · A per-component virtual age V · ANY: q dial spans minimal ↔ renewal · Statistical (with CIs) · 0.838 at the chosen q = 0.5 (4,000 h)
- ⑪ Choosing the method is choosing the repair physics — Method 1: minimal repair; Methods 2 and 3: renewal; Method 4: any q.

METHOD 1 SELF-CHECK — QUASI-STEADY SHORTCUT

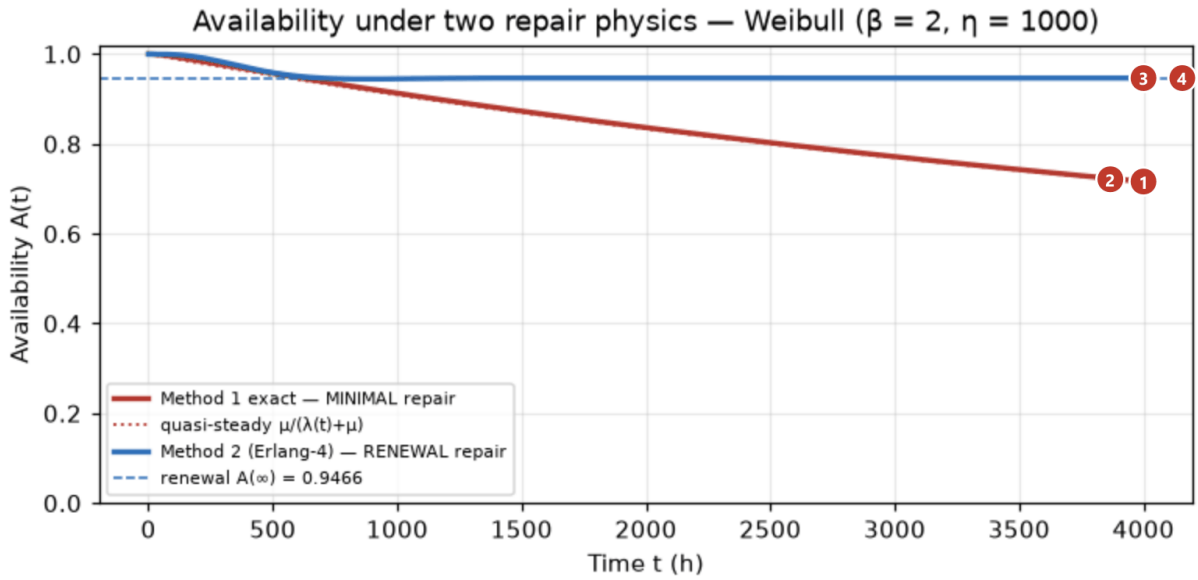
1 At mission: exact 0.7161 vs quasi-steady 0.7143.

2 A(t) ≈ μ/(λ(t)+μ) once the chain settles (~150 h). Largest deviation from the exact ODE after settling: **0.0047** — the hazard moves slowly against the repair timescale, the integration self-checks, and the shortcut is engineering-grade here.

3 Every Method-1 figure assumes minimal repair (as bad as old). If the machine is replaced at failure, use Method 3; Method 1 is pessimistic by 7.09× at 4,000 h.

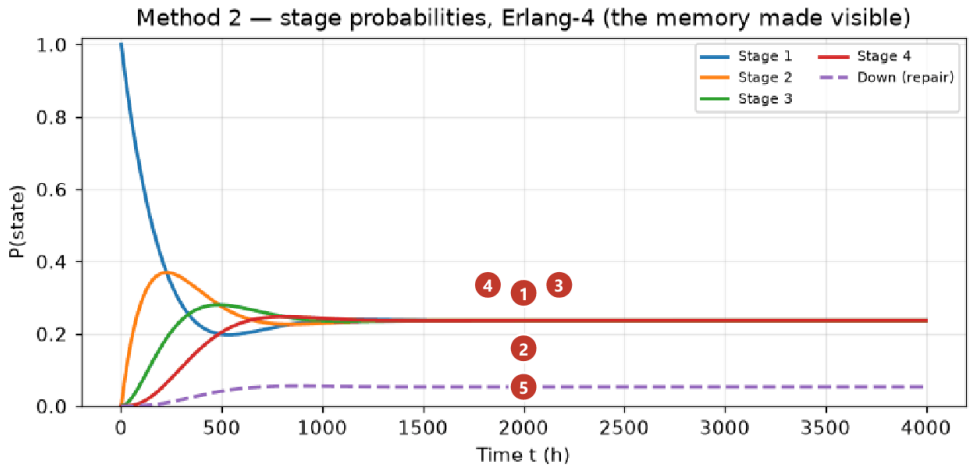
Method 1 self-check — quasi-steady shortcut

- ① At mission: exact 0.7161 vs quasi-steady 0.7143.
- ② A(t) ≈ μ/(λ(t)+μ) once the chain settles (~150 h). Largest deviation from the exact ODE after settling: 0.0047 — the hazard moves slowly against the repair timescale, the integration self-checks, and the shortcut is engineering-grade here.
- ③ Every Method-1 figure assumes minimal repair (as bad as old). If the machine is replaced at failure, use Method 3; Method 1 is pessimistic by 7.09× at 4,000 h. The 7.09 is the hazard rate at 4,000 h times the MTTF, λ(4000) × MTTF = 0.008 × 886.23: how many times faster the worn gearbox fails than its average over a whole life. It is not the ratio of the two methods' unavailabilities.



Plot

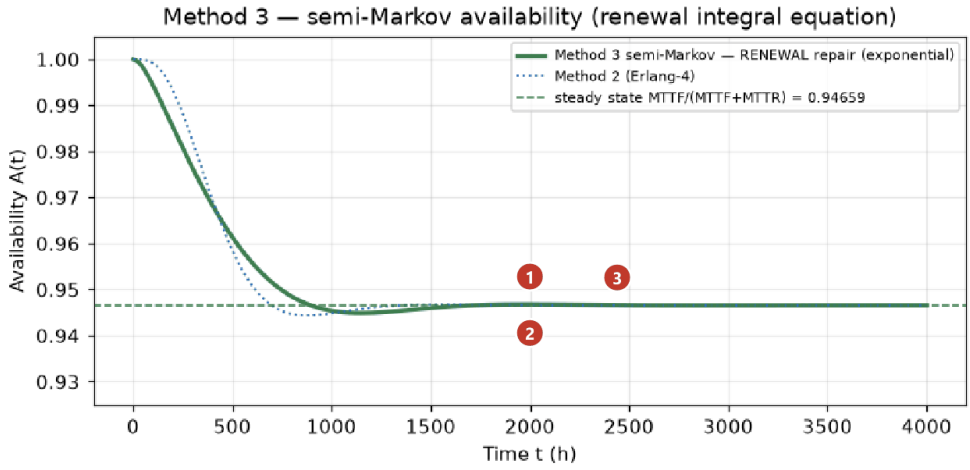
- ① The line: Method 1 exact — MINIMAL repair.
- ② The line: quasi-steady μ/(λ(t)+μ).
- ③ The line: Method 2 (Erlang-4) — RENEWAL repair.
- ④ The horizontal line: renewal A(∞) = 0.9466.



Interactive Plot

Plot

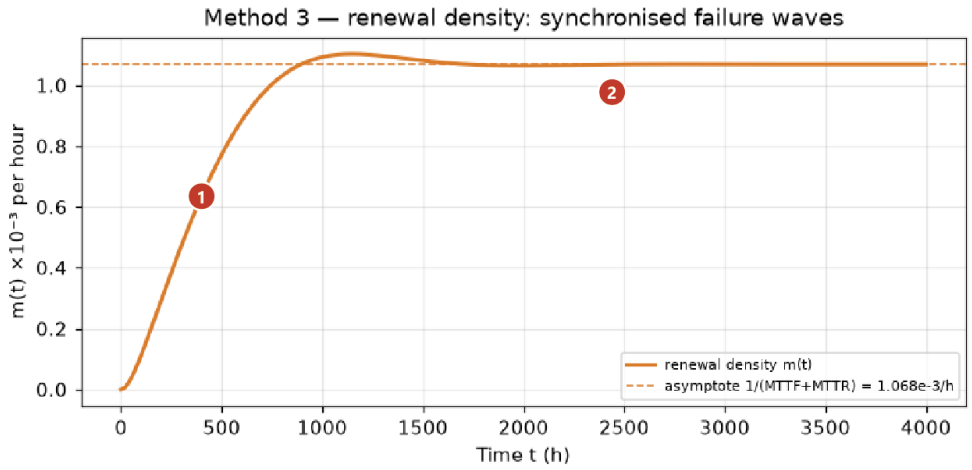
- ① The line: Stage 1.
- ② The line: Stage 2.
- ③ The line: Stage 3.
- ④ The line: Stage 4.
- ⑤ The line: Down (repair).



Interactive Plot

Plot

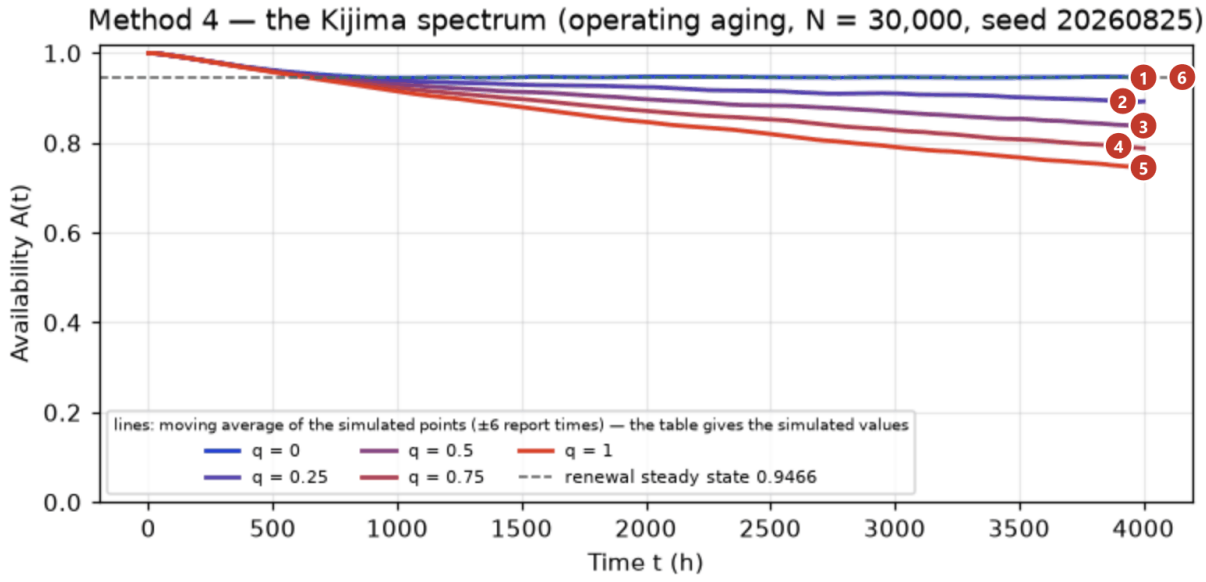
- ① The line: Method 3 semi-Markov — RENEWAL repair (exponential).
- ② The line: Method 2 (Erlang-4).
- ③ The horizontal line: steady state MTTF/(MTTF+MTTR) = 0.94659.



Interactive Plot

Plot

- ① The line: renewal density m(t).
- ② The horizontal line: asymptote 1/(MTTF+MTTR) = 1.068e-3/h.



Plot

- ① The line: q = 0.
- ② The line: q = 0.25.
- ③ The line: q = 0.5.
- ④ The line: q = 0.75.
- ⑤ The line: q = 1.
- ⑥ The horizontal line: renewal steady state 0.9466.

STAGE OCCUPANCY — ERLANG-4 STAGE MODEL OF WEIBULL (B = 2, H = 1000)

1 The stage probabilities ARE the model's memory of the machine's age — wear made visible. Each stage hops at ν = 0.004514/h.

2 State	3 P(state) at mission	4 Steady-state π
5 Stage 1	0.236649	0.236649
6 Stage 2	0.236649	0.236649
7 Stage 3	0.236649	0.236649
8 Stage 4	0.236649	0.236649
9 Down (repair)	0.053406	0.053406

Stage occupancy — Erlang-4 stage model of Weibull (β = 2, η = 1000)

- ① The stage probabilities ARE the model's memory of the machine's age — wear made visible. Each stage hops at ν = 0.004514/h.
- ② State
- ③ P(state) at mission
- ④ Steady-state π
- ⑤ Stage 1 · 0.236649 · 0.236649
- ⑥ Stage 2 · 0.236649 · 0.236649
- ⑦ Stage 3 · 0.236649 · 0.236649
- ⑧ Stage 4 · 0.236649 · 0.236649
- ⑨ Down (repair) · 0.053406 · 0.053406

METHOD 2 TRANSIENT — ERLANG-4 STAGE CHAIN, REPAIR ARROW TO STAGE 1

1 Erlang-4 matches the mean exactly and the spread to $CV^2 = 0.25$ against the target 0.2732 — the transient carries that error; the steady state does not.

2 t (h)	3 A(t)	4 Stage 1	5 Stage 2	6 Stage 3	7 Stage 4	8 Down (repair)
9 250	0.9882	0.3365	0.3677	0.2064	0.0775	0.0118
10 500	0.9585	0.1986	0.2752	0.2801	0.2046	0.0415
11 1,000	0.9448	0.2362	0.2295	0.2360	0.2431	0.0552
12 2,000	0.9466	0.2365	0.2366	0.2368	0.2367	0.0534
13 4,000	0.9466	0.2366	0.2366	0.2366	0.2366	0.0534

14 A(∞) = 0.94659; corrective failures 9.36 per year (π_Stage 4 · ν · 8760 h); ν = 0.004514/h. The stage probabilities are plotted above.

Method 2 transient — Erlang-4 stage chain, repair arrow to Stage 1

- ① Erlang-4 matches the mean exactly and the spread to $CV^2 = 0.25$ against the target 0.2732 — the transient carries that error; the steady state does not.
- ② t (h)
- ③ A(t)
- ④ Stage 1
- ⑤ Stage 2
- ⑥ Stage 3
- ⑦ Stage 4
- ⑧ Down (repair)
- ⑨ 250 · 0.9882 · 0.3365 · 0.3677 · 0.2064 · 0.0775 · 0.0118
- ⑩ 500 · 0.9585 · 0.1986 · 0.2752 · 0.2801 · 0.2046 · 0.0415
- ⑪ 1,000 · 0.9448 · 0.2362 · 0.2295 · 0.2360 · 0.2431 · 0.0552
- ⑫ 2,000 · 0.9466 · 0.2365 · 0.2366 · 0.2368 · 0.2367 · 0.0534
- ⑬ 4,000 · 0.9466 · 0.2366 · 0.2366 · 0.2366 · 0.2366 · 0.0534
- ⑭ A(∞) = 0.94659; corrective failures 9.36 per year (π_Stage 4 · ν · 8760 h); ν = 0.004514/h. The stage probabilities are plotted above.

TRUST CHECK — ERLANG-4 STAGE MODEL OF WEIBULL (B = 2, H = 1000)

1 The simulation replays the same chain event by event with a fixed seed (deterministic) — an independent surface, not a re-run of the formula.

2 Analytic A(t = 4000) = **0.946594** · independent seeded simulation = **0.9425 ± 0.0072** (95%, 4,000 histories) — the analytic answer lies inside the simulation interval; two independent computations agree.

Trust check — Erlang-4 stage model of Weibull (β = 2, η = 1000)

- ① The simulation replays the same chain event by event with a fixed seed (deterministic) — an independent surface, not a re-run of the formula.
- ② Analytic A(t = 4000) = 0.946594 · independent seeded simulation = 0.9425 ± 0.0072 (95%, 4,000 histories) — the analytic answer lies inside the simulation interval; two independent computations agree.

METHOD 3 TRANSIENT — SEMI-MARKOV, RENEWAL EQUATION (EXPONENTIAL, MTTR 50 H)

1 One cycle = one life + one repair; m(t) = g(t) + ∫g(t−u)m(u)du and A(t) = R(t) + ∫m(u)R(t−u)du, with the cycle density built mass-preserving from exact CDF differences.

2 t (h)	3 A(t) — Method 3 semi-Markov	4 Method 2 Erlang-4	5 Method 1 minimal repair
6 250	0.9807	0.9882	0.9803
7 500	0.9614	0.9585	0.9568
8 1,000	0.9454	0.9448	0.9129
9 2,000	0.9468	0.9466	0.8363
10 4,000	0.9466	0.9466	0.7161

11 Grid step 0.25 h (16,000 cells, Richardson-extrapolated; largest correction 1.0e-04); cycle mass 1.000000000. Steady state MTTF/(MTTF+MTTR) = 0.94659 — exact for any repair law with this mean.

Method 3 transient — semi-Markov, renewal equation (exponential, MTTR 50 h)

- ① One cycle = one life + one repair; m(t) = g(t) + ∫g(t−u)m(u)du and A(t) = R(t) + ∫m(u)R(t−u)du, with the cycle density built mass-preserving from exact CDF differences.
- ② t (h)
- ③ A(t) — Method 3 semi-Markov
- ④ Method 2 Erlang-4
- ⑤ Method 1 minimal repair
- ⑥ 250 · 0.9807 · 0.9882 · 0.9803
- ⑦ 500 · 0.9614 · 0.9585 · 0.9568
- ⑧ 1,000 · 0.9454 · 0.9448 · 0.9129
- ⑨ 2,000 · 0.9468 · 0.9466 · 0.8363
- ⑩ 4,000 · 0.9466 · 0.9466 · 0.7161
- ⑪ Grid step 0.25 h (16,000 cells, Richardson-extrapolated; largest correction 1.0e-04); cycle mass 1.000000000. Steady state MTTF/(MTTF+MTTR) = 0.94659 — exact for any repair law with this mean.

RENEWAL DENSITY M(T) — THE FAILURE WAVES

1 Synchronised first failures return from repair around one mean life (886 h) and damp toward the long-run cycle rate 1.068e-3/h.

2 t (h)	3 m(t) $\times 10^{-3}$ per hour
500	0.7724
886 (MTTF)	1.0659
1,000	1.0924
1,500	1.0805
2,000	1.0641
3,000	1.0684
4,000	1.0681

4 Asymptote 1/(MTTF + MTTR) = 1.068e-3/h — a fleet commissioned together sends workload waves to the repair shop at multiples of the mean life until de-phasing washes them out.

Renewal density m(t) — the failure waves

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- ④ Asymptote 1/(MTTF + MTTR) = 1.068e-3/h — a fleet commissioned together sends workload waves to the repair shop at multiples of the mean life until de-phasing washes them out.

METHOD 4 VALIDATION — THE SIMULATOR AGAINST THE EXACT METHODS

1 Validated: q = 0 reproduces Method 3 and q = 1 (calendar aging) reproduces Method 1 within the 95 % intervals at every report time.

2 t (h)	3 Simulated q = 0 (operating) ± 95 %	4 Method 3 exact	5 Simulated q = 1 (calendar) ± 95 %	6 Method 1 exact
7 250	0.9812 ± 0.0015	0.9807	0.9811 ± 0.0015	0.9803
8 500	0.9621 ± 0.0022	0.9614	0.9574 ± 0.0023	0.9568
9 1,000	0.9445 ± 0.0026	0.9454	0.9105 ± 0.0032	0.9129
10 2,000	0.9471 ± 0.0025	0.9468	0.8352 ± 0.0042	0.8363
11 4,000	0.9457 ± 0.0026	0.9466	0.7160 ± 0.0051	0.7161

12 N = 30,000 histories per run, seed 20260825; 95 % intervals with the rule-of-three floor at the extremes. The simulator earns interior answers only by reproducing both exact methods at its extremes.

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- ⑫ N = 30,000 histories per run, seed 20260825; 95 % intervals with the rule-of-three floor at the extremes. The simulator earns interior answers only by reproducing both exact methods at its extremes.

METHOD 4 SPECTRUM — RESTORATION Q = 0 … 1 (OPERATING AGING), THE DECISION VIEW

1. Repair quality changes the breakdown rate 3.0× across q = 0 … 1; moving from q = 0.5 to q = 0 saves 8.7 failures per year.

2 Restoration q	3 A(4,000 h) ± 95 %	4 Failures per year ± SE
5 0	0.9457 ± 0.0026	8.65 ± 0.01
6 0.25	0.8925 ± 0.0035	12.90 ± 0.02
7 0.5 (chosen)	0.8383 ± 0.0042	17.36 ± 0.03
8 0.75	0.7877 ± 0.0046	21.69 ± 0.03
9 1	0.7451 ± 0.0049	25.77 ± 0.03

10. Calendar aging vs operating aging at q = 1: 0.716 vs 0.745 at 4,000 h.

11. Kijima I: after each repair V ← V + q·X (operating clock) — the bay time is added to V on the calendar clock; q = 0 as good as new, q = 1 as bad as old. Price the failures saved against the overhaul premium.

Method 4 spectrum — restoration q = 0 … 1 (operating aging), the decision view

- ① Repair quality changes the breakdown rate 3.0× across q = 0 … 1; moving from q = 0.5 to q = 0 saves 8.7 failures per year.
- ② Restoration q
- ③ A(4,000 h) ± 95 %
- ④ Failures per year ± SE
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The pump (constant rates), after step 30:

INPUT DATA

1	Model	preset — 1 unit repairable (λ = 0.002, μ = 0.05)
2	States	2 (Up, Down)
3	Up (available) states	Up
4	Initial state	Up
5	Mission time t	10000
6	Operating hours / yr	8760

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- ③ Up (available) states Up.
- ④ Initial state Up.
- ⑤ Mission time t 10000. Operating hours / yr 8760.
- ⑥ Operating hours / yr 8760.

RESULTS

1	Steady-state availability A(∞)	0.961538
2	Mission availability A(t)	0.961538
3	Mission reliability R(t)	0.000000
4	MTTF (first entry into Down)	500.0
5	Expected downtime / yr at A(∞)	336.9 h

Results

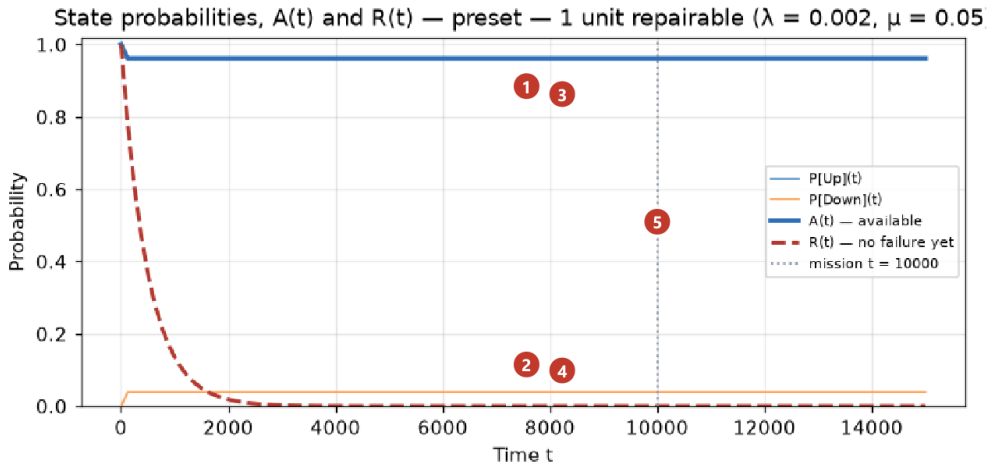
- ① Steady-state availability A(∞) 0.961538. In the long run the pump is up 0.961538 of the time.
- ② Mission availability A(t) 0.961538. At 10,000 hours it has settled at A(∞).
- ③ Mission reliability R(t) 0.000000. The pump is almost sure to have failed at least once in 10,000 hours.
- ④ MTTF (first entry into Down) 500.0. ⑤ Expected downtime / yr at A(∞) 336.9 h.
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INTERPRETATION

1 Starting from 'Up', the system is available at t = 10000 with probability 0.961538; the probability it has NEVER been down by then is 0.000000. Availability counts repaired returns to service; reliability does not — the gap between the two curves is what the repair crew buys.

Interpretation

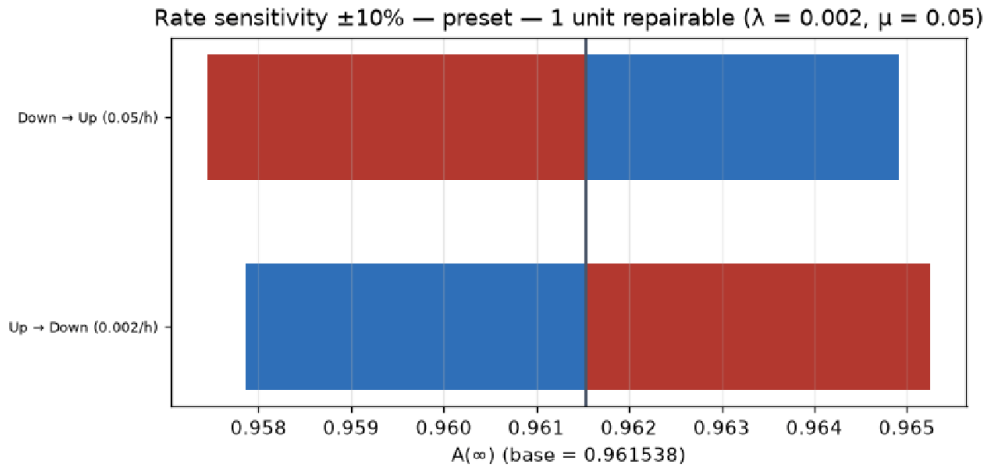
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Interactive Plot

Plot

- ① The line: P[Up](t).
- ② The line: P[Down](t).
- ③ The line: A(t) — available.
- ④ The line: R(t) — no failure yet.
- ⑤ The vertical line: mission t = 10000.



Interactive Plot

Plot

- The rate sensitivity: A(∞) when each rate moves by ±10%.

STATES AT MISSION AND STEADY STATE — PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05)

1 First state is the initial state. π solves πQ = 0 with Σπ = 1; for a chain with an absorbing failed class, π concentrates there.

2 State	3 Class	4 P(state) at mission t	5 Steady-state π
6 Up	Up	0.961538	0.961538
7 Down	Down	0.038462	0.038462

States at mission and steady state — preset — 1 unit repairable (λ = 0.002, μ = 0.05)

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- ② State
- ③ Class
- ④ P(state) at mission t
- ⑤ Steady-state π
- ⑥ Up · Up · 0.961538 · 0.961538
- ⑦ Down · Down · 0.038462 · 0.038462

TRANSITION RATES — PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05)

1 Generator convention: Q[i][j] = rate i→j, diagonal = −(row sum).

2 From	3 To	4 Rate (/h)	5 Mean stay 1/rate (h)
6 Up	Down	0.002	500.0
7 Down	Up	0.05	20.0

Transition rates — preset — 1 unit repairable (λ = 0.002, μ = 0.05)

- ① Generator convention: Q[i][j] = rate i→j, diagonal = −(row sum).
- ② From
- ③ To
- ④ Rate (/h)
- ⑤ Mean stay 1/rate (h)
- ⑥ Up · Down · 0.002 · 500.0
- ⑦ Down · Up · 0.05 · 20.0

AVAILABILITY VS RELIABILITY — HOW TO READ THIS ANALYSIS

1 Not detected ≠ proven absent: a Markov model assumes CONSTANT rates — if failure data show wear-out (Weibull β > 1), treat these numbers as approximate.

2 **A(t)** is the probability the system is UP at instant t — failures repaired before t still count as available. **R(t)** is the probability of ZERO system failures up to t (Down states made absorbing). For non-repairable models the two are identical; with repair, A(t) settles at A(∞) while R(t) keeps falling toward 0. Use A(∞) for downtime budgets, R(t) for mission-critical 'must not fail' questions, MTTF for the average first failure.

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DESIGNER 1 — FIX FASTER (PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05))

1 Exact inversion of A(∞) in μ (2-state closed form μ_req = A·λ/(1−A); multi-state numerically).

2 Current steady availability **0.961538** misses the target 99.00%. Keeping this structure, repairs must speed up to μ = 0.198/h — i.e. **finish within 5.05 h on average** (currently no target-meeting margin at the entered μ).

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DESIGNER 2 — ADD UNITS (HOT-PARALLEL LADDER, PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05))

1 ★ first n meeting the target 99.00% — shared crew: n* = 2 · dedicated crews: n* = 2. Marginal gains shrink with every unit; buy the cheaper of the two levers (faster repair vs another unit).

2 Units n	3 A(∞) shared repair	4 A(∞) dedicated repair
5 1	0.961538	0.961538
6 2	0.997046 ★	0.998521 ★
7 3	0.999660	0.999943
8 4	0.999948	0.999998
9 5	0.999990	1.000000
10 6	0.999998	1.000000

Designer 2 — add units (hot-parallel ladder, preset — 1 unit repairable (λ = 0.002, μ = 0.05))

- ① ★ first n meeting the target 99.00% — shared crew: n* = 2 · dedicated crews: n* = 2. Marginal gains shrink with every unit; buy the cheaper of the two levers (faster repair vs another unit).
- ② Units n
- ③ A(∞) shared repair
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- ⑩ 6 · 0.999998 · 1.000000

REPAIR POLICY — SHARED VS DEDICATED (2-PARALLEL AT THESE RATES)

1 The second crew buys 12.92 h/yr of uptime = 129,203/yr at the entered production loss — compare with the crew's cost to decide.

2 Policy	3 A(∞)	4 Downtime h/yr
5 Shared (1 crew)	0.997046	25.9
6 Dedicated (2 crews)	0.998521	13.0

Repair policy — shared vs dedicated (2-parallel at these rates)

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- ② Policy
- ③ A(∞)
- ④ Downtime h/yr
- ⑤ Shared (1 crew) · 0.997046 · 25.9
- ⑥ Dedicated (2 crews) · 0.998521 · 13.0

DOWNTIME ECONOMICS — PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05)

1 Cost = π × 8760 h/yr × 10,000/hr; total 3,369,231/yr at steady state. For an absorbing chain π sits in the failed class — read the total as the eventual all-down rate, not an operating budget.

2 Down state	3 π (share of time)	4 Hours / yr	5 Cost / yr	6 Share of downtime cost
7 Down	0.038462	336.9	3,369,231	100.0%

Downtime economics — preset — 1 unit repairable (λ = 0.002, μ = 0.05)

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- ② Down state
- ③ π (share of time)
- ④ Hours / yr
- ⑤ Cost / yr
- ⑥ Share of downtime cost
- ⑦ Down · 0.038462 · 336.9 · 3,369,231 · 100.0%

TRUST CHECK — PRESET — 1 UNIT REPAIRABLE (Λ = 0.002, M = 0.05)

1 The simulation replays the same chain event by event with a fixed seed (deterministic) — an independent surface, not a re-run of the formula.

2 Analytic A(t = 10000) = **0.961538** · independent seeded simulation = **0.9605 ± 0.0060** (95%, 4,000 histories) — the analytic answer lies inside the simulation interval; two independent computations agree.

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Part 5 — How to interpret the results

The answer depends on what a repair does. The gearbox's mean life is 886.23 hours. If every repair renews it, its availability settles at 0.94659 - the mean life over the mean life plus the mean repair, 886.23 / 936.23 - and Methods 2 and 3 both give 0.9466 at 4,000 hours. If every repair only patches the failed part and leaves the gearbox as old as it was, its availability at 4,000 hours is 0.7161 and still falling: 0.5006 by 10,000 hours. Neither answer is "more correct"; read the line that matches your workshop.

Patching is a losing race on a wearing machine. Under minimal repair the failure rate keeps growing while the repair rate stays the same, so availability never settles. The quick estimate μ / (λ(t) + μ) = 0.02 / 0.028 = 0.7143 at 4,000 hours is close to the exact 0.7161: the hazard moves slowly against the repair time.

A repair in between. Method 4 spans the two: q = 0 renews the gearbox, q = 1 patches it. Trust its numbers only when its validation card reads Validated: q = 0 reproduces Method 3 and q = 1 (calendar aging) reproduces Method 1 within the 95 % intervals at every report time. Its spectrum then prices repair quality - how many failures a year a better repair saves - against what the better repair costs.

What to plan the repair crew for. With renewal, the gearbox fails 9.36 times a year in the long run: one failure per mean life plus mean repair, 8,760 / 936.23 hours.

The constant-rate pump. In the long run the pump is up 0.961538 of the time - 0.05 / (0.002 + 0.05) - and down 336.9 hours a year. That misses the 99% target: the designer cards give the two ways to reach it, faster repairs or more units, and the economics card prices the downtime. Its mission reliability over 10,000 hours is 0.000000: with a mean time to failure of 500 hours, the pump is almost sure to fail at least once. Use A(∞) for downtime budgets and R(t) only for "must not fail" missions.

Constant rates are an assumption. If the pump's failure data show wear-out (Weibull $\beta > 1$), model it as the gearbox: the constant-rate numbers are then approximate.