Part 1 — What the function does

What it calculates. A repair rarely makes a machine as good as new, and it rarely leaves it exactly as bad as before. A pump with a new seal is better than before the failure, but its bearings and impeller are still old. This analysis measures where your repairs sit between those two extremes, from the failure log of a fleet of like machines. It fits a generalised renewal process (GRP): a Weibull hazard with shape β and scale η, run on a virtual age that each repair resets in part. The restoration factor q says how much of the age a repair leaves.

q	What a repair does
0	Full renewal: the machine is as good as new after each repair
Between 0 and 1	Partial restoration: with Kijima Type I, q = 0.2 leaves 20 % of the age gained since the last repair
1	Minimal repair: the machine is as old as it was just before the failure

Term	Meaning
Virtual age V	The age the hazard sees. Kijima Type I: V ← V + q·X, where X is the time since the last repair. Kijima Type II: V ← q·(V + X)
Profile ℓ(q)	The best log-likelihood at each fixed q, with β and η fitted. Its maximum ℓ_max gives q̂
Profile interval	The q values whose ℓ(q) lies above ℓ_max minus half the chi-square value at your confidence level

What ReliaEn prints	What it tells you
β̂, η̂ and q̂ with their intervals	How the machines age, and how much of that age a repair leaves
Two likelihood-ratio tests, against q = 0 and q = 1	Whether the repairs differ from a full renewal and from a minimal repair
MTBF at the mean virtual age	The mean time to the next failure of a unit at the fleet's mean virtual age after each unit's last repair
The profile of ℓ(q), as a plot and a table	How sharply the data pin q down
A Bayesian card for q	One range for q, from a uniform prior on 0 to 1
A fit check	The failures the fit expects against the failures on record, beside the two extremes
A forward card	The failures to expect over a horizon you choose; with an MTTR the availability, with costs the best overhaul interval

From which data. One row per failure: the unit's id and the event time, on the unit's own clock - cumulative from clock 0, or the gap since its previous failure - with the end of observation for the fleet or per unit.

When an engineer uses it. When several machines of one type are repaired again and again, the time of every failure and the end of observation are on record, and the question is how good the repairs are.

When not to use it.

Situation	Use instead
The log of one machine, and whether it is improving or wearing out	Life Data ▸ Estimate Failure Function ▸ Repairable unit data… (chapter 3.1)
The mean cumulative repairs of a fleet, without a model	Life Data ▸ Repairable Fleet… (chapter 3.3)
Every failed item is replaced by a new one, and no machine has a second failure on record	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution… (chapter 2.6)

Part 2 — The example's input data

Setting. Four boiler feed pumps were each watched for 5,000 operating hours and failed 40 times between them. The maintenance planner wants to know how much a repair restores a pump.

Pump	Gaps since the previous failure, hours
1	533.4, 688.2, 623.4, 498.2, 1246.9, 750.0, 96.0, 34.7, 42.7
2	209.0, 1338.7, 506.3, 465.0, 262.4, 605.9, 190.2, 116.0, 109.1, 551.1, 355.0
3	1084.3, 191.2, 534.2, 84.0, 255.9, 890.3, 148.8, 69.3, 315.6, 296.2, 688.5
4	654.7, 461.1, 724.2, 766.6, 195.2, 14.1, 41.3, 607.3, 1232.9

Input	Value
Entry	Gap since the previous event
Observation end (fleet)	5000
Kijima type / Confidence level %	Type I (V ← V + q·X) / 95
Triple / Horizon (h)	from this fit (β̂, η̂, q̂) / 8760
File in the worked-example zip	3-GRP-Kijima/Example-1.csv — columns Unit, Gap, 40 rows

	P	Q	R	S	T	① U	② V	W
1						1	533.4	
2						1	688.2	
3						1	623.4	
4						1	498.2	
5						1	1246.9	
6						1	750.0	
7						1	96.0	
8						1	34.7	
9						1	42.7	
10						2	209.0	
11						2	1338.7	
12						2	506.3	
13						2	465.0	
14						2	262.4	
15						2	605.9	
16						2	190.2	
17						2	116.0	
18						2	109.1	
19						2	551.1	
20						2	355.0	
21						3	1084.3	
22						3	191.2	
23						3	534.2	
24						3	84.0	
25						3	255.9	
26						3	890.3	
27						3	148.8	
28						3	69.3	
29						3	315.6	
30						3	296.2	
31						3	688.5	
32						4	654.7	

The example data in the worksheet

- ① Column U: the pump of each failure.
- ② Column V: the gap before it.

Part 3 — How to use it, step by step

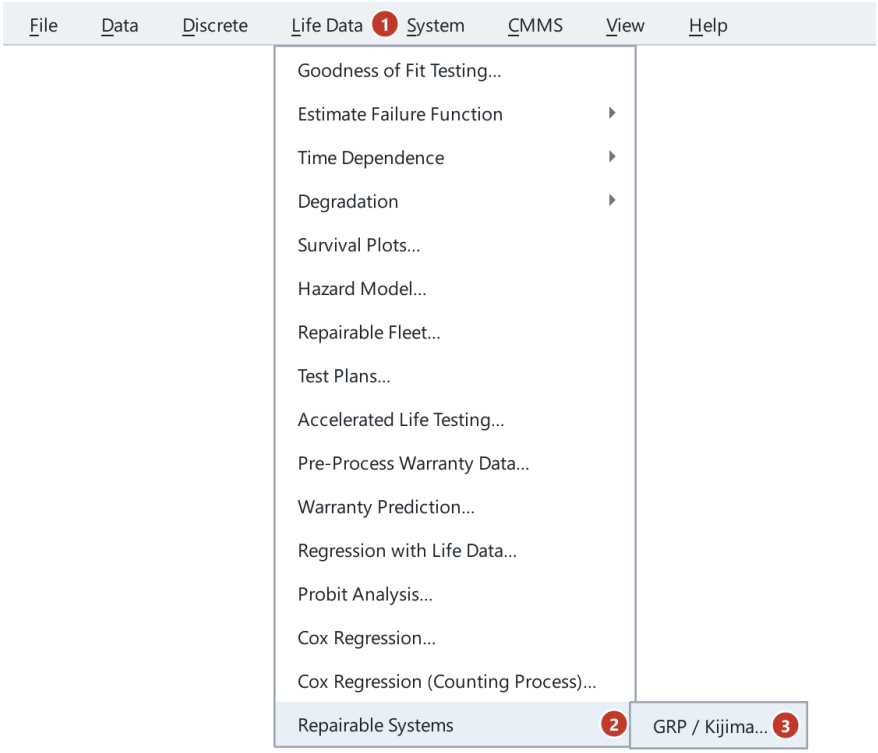
1. **Put the failure log in columns U and V.** Copy the 40 rows of Example-1.csv - its columns Unit and Gap - into columns U and V of the worksheet, starting at row 1. Any free columns will do; the form names them in steps 4 and 5. The picture shows the first 32 rows.

	P	Q	R	S	T	① U	② V	W
1						1	533.4	
2						1	688.2	
3						1	623.4	
4						1	498.2	
5						1	1246.9	
6						1	750.0	
7						1	96.0	
8						1	34.7	
9						1	42.7	
10						2	209.0	
11						2	1338.7	
12						2	506.3	
13						2	465.0	
14						2	262.4	
15						2	605.9	
16						2	190.2	
17						2	116.0	
18						2	109.1	
19						2	551.1	
20						2	355.0	
21						3	1084.3	
22						3	191.2	
23						3	534.2	
24						3	84.0	
25						3	255.9	
26						3	890.3	
27						3	148.8	
28						3	69.3	
29						3	315.6	
30						3	296.2	
31						3	688.5	
32						4	654.7	

Step 1 — the failure log in columns U and V

- ① Column U: the pump of each failure, 1 to 4.
- ② Column V: the gap before it, in hours.

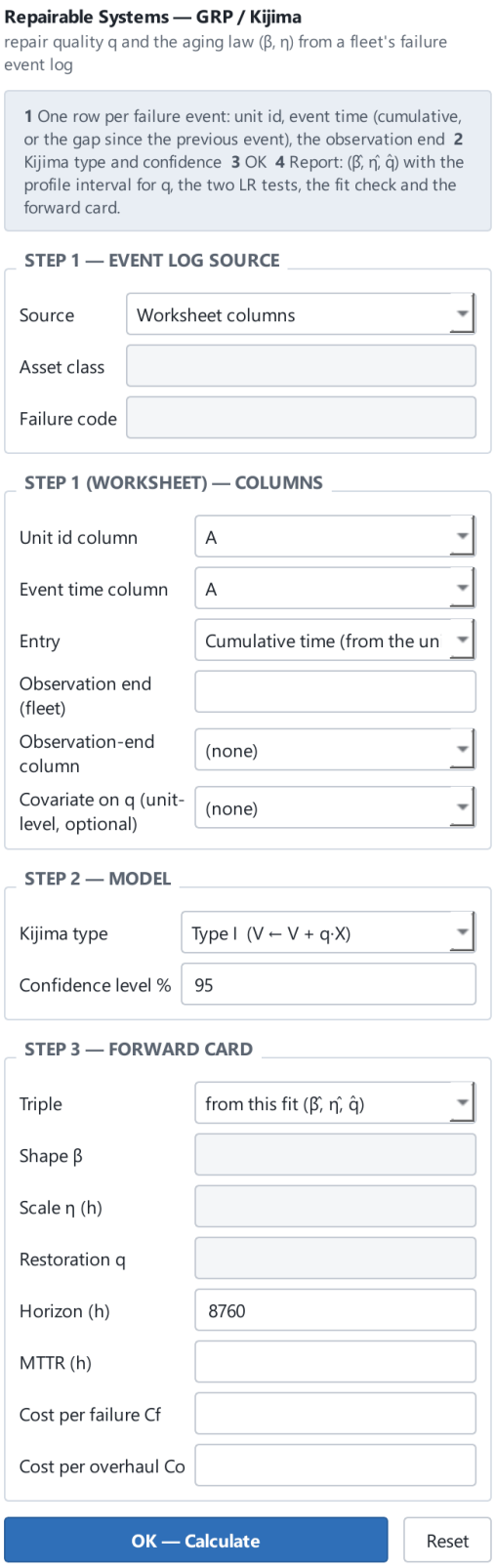
2. **Choose Life Data ▸ Repairable Systems ▸ GRP / Kijima…** in the menu bar. The form Repairable Systems — GRP / Kijima opens in the left panel.



Step 2 — the menu path

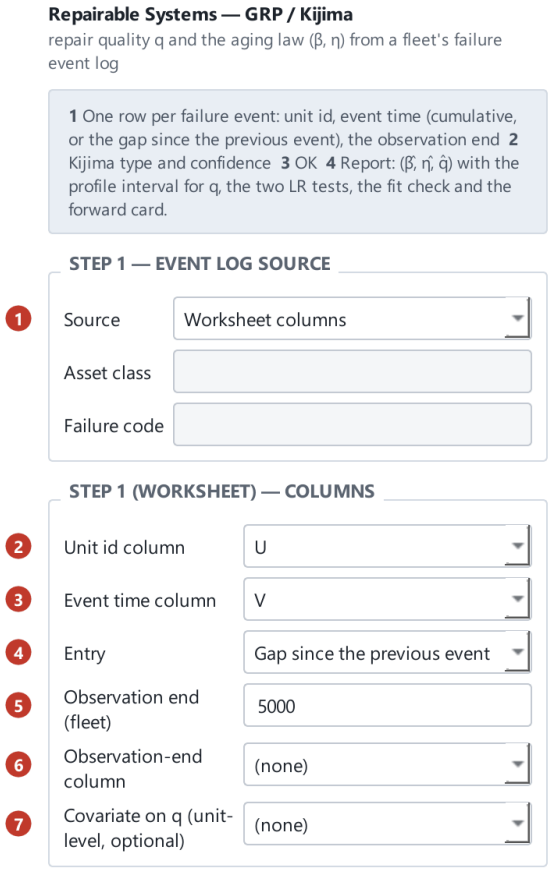
- ① Life Data: the menu.
- ② Repairable Systems: the submenu.
- ③ GRP / Kijima…: the entry to click.

The form as it opens, before any box is set:

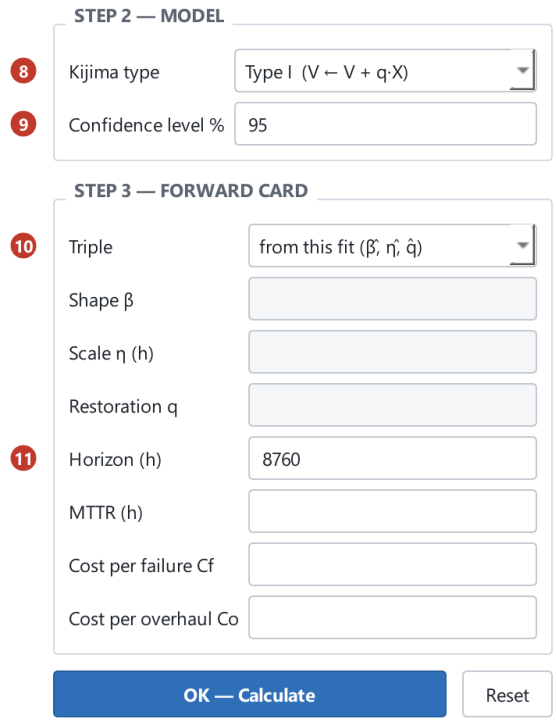


Step 2 — the form as it opens

Steps 3 to 13 set or check the form's eleven boxes, shown in two pictures - ① to ⑦, then ⑧ to ⑪:



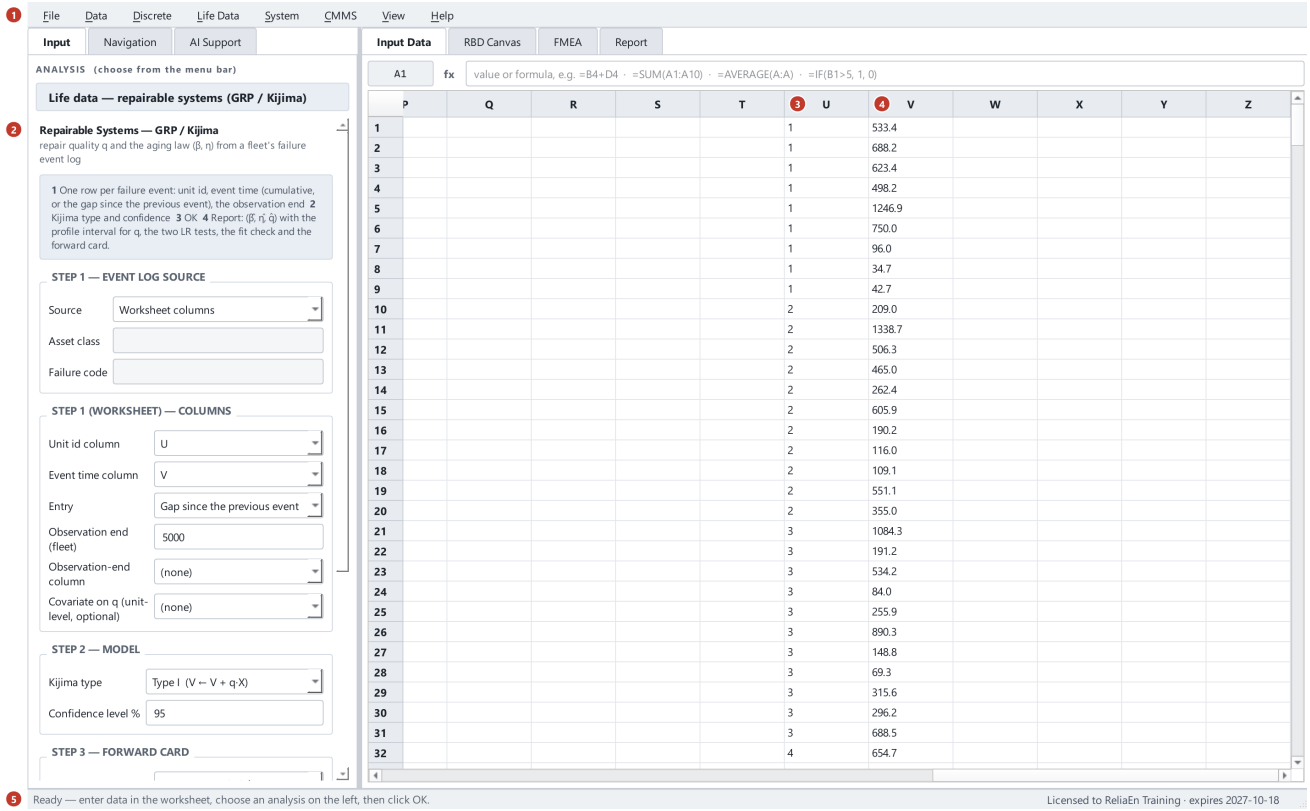
Steps 3 to 9 — the source and column boxes set for this example



Steps 10 to 13 — the model and forward-card boxes

3. ① **Check that Source reads Worksheet columns.** Asset class and Failure code stay grey; they are for the CMMS route.
4. ② **In Unit id column, choose U.** The box opens on A.
5. ③ **In Event time column, choose V.** The box opens on A.
6. ④ **In Entry, choose Gap since the previous event.** The box opens on Cumulative time (from the unit's clock 0); the file holds gaps.
7. ⑤ **In Observation end (fleet), type 5000.** Every pump was watched for 5,000 hours.
8. ⑥ **Check that Observation-end column reads (none).** It is for one end per unit.
9. ⑦ **Check that Covariate on q (unit-level, optional) reads (none).**
10. ⑧ **Check that Kijima type reads Type I (V ← V + q·X).**
11. ⑨ **Check that Confidence level % reads 95.**
12. ⑩ **Check that Triple reads from this fit (β̂, η̂, q̂).**
13. ⑪ **Check that Horizon (h) reads 8760.** One year. MTTR (h) and the two cost boxes stay empty in this example.

14. **Click OK — Calculate** at the foot of the form. ReliaEn calculates and shows the report.



Step 14 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 13; it is taller than the panel, so OK — Calculate is below the part in view.
- ③ Column U: the pumps.
- ④ Column V: the gaps.
- ⑤ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — Repairable Systems (GRP / Kijima)

2026-11-04 11:36:48

$\ell = \Sigma_{events}\ [\ln \lambda(V + X) - H(V + X) + H(V)] - \Sigma_{tails}\ [H(V + X_c) - H(V)]$; $H(t) = (t/\eta)^{\beta}$; Type I $V \leftarrow V + q \cdot X$, Type II $V \leftarrow q \cdot (V + X)$

The report's head

- The title Life Data — Repairable Systems (GRP / Kijima), the date and time of the run, and the likelihood of the model in one line.

INPUT DATA

1	Data	this worksheet — 4 units, 40 failures, 4 censored runs, observation 5,000 h (fleet)
2	Model	Weibull hazard + Kijima Type I virtual age V ← V + q·X
3	Clock	clock starts at 0 for every unit
4	Confidence level	95 %

Input data

- ① Data this worksheet — 4 units, 40 failures, 4 censored runs, observation 5,000 h (fleet): after each pump's last failure, a censored run up to 5,000 hours.
- ② Model Weibull hazard + Kijima Type I virtual age V ← V + q·X.
- ③ Clock clock starts at 0 for every unit.
- ④ Confidence level 95 %.

RESULTS

1	Shape β̂	1.8975 (SE 0.3466) [1.3265, 2.7142]
2	Scale η̂ (h)	785.56 (SE 144.20) [548.19, 1,125.71]
3	Restoration q̂	0.1362 — 95 % profile interval [0.015, 0.593]
4	ℓ_max (profile maximum)	-284.3482
5	LR vs renewal (q = 0)	5.335 (p 0.0209)
6	LR vs minimal repair (q = 1)	4.990 (p 0.0255)
7	MTBF at the mean virtual age (h)	358.70 (V̄ 629.21 h)

Results

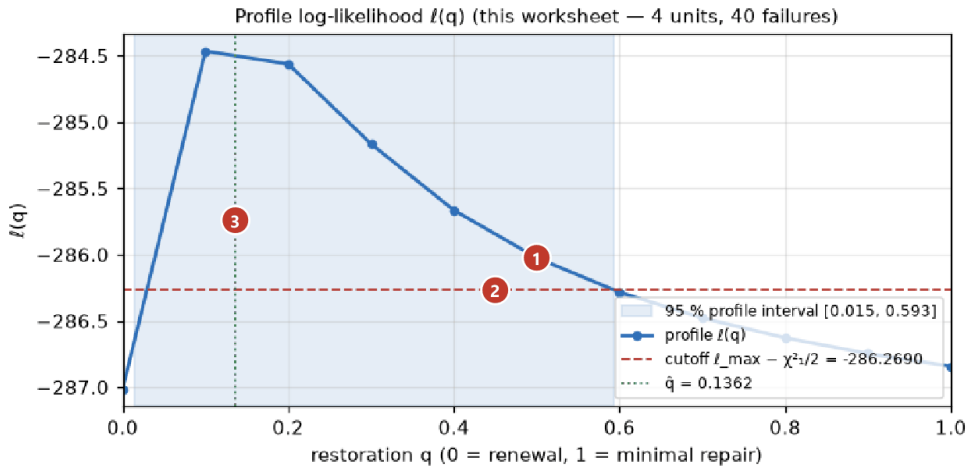
- ① Shape β̂ 1.8975 (SE 0.3466) [1.3265, 2.7142].
- ② Scale η̂ (h) 785.56 (SE 144.20) [548.19, 1,125.71].
- ③ Restoration q̂ 0.1362 — 95 % profile interval [0.015, 0.593].
- ④ ℓ_max (profile maximum) -284.3482.
- ⑤ LR vs renewal (q = 0) 5.335 (p 0.0209).
- ⑥ LR vs minimal repair (q = 1) 4.990 (p 0.0255).
- ⑦ MTBF at the mean virtual age (h) 358.70 (V̄ 629.21 h).

INTERPRETATION

1 Repairs preserve about 86 % of the unit's health (q = 0 would be full renewal, q = 1 no repair at all). The 95 % profile interval for q is [0.015, 0.593] (width 0.578); q = 0 lies outside it and q = 1 lies outside it.

Interpretation

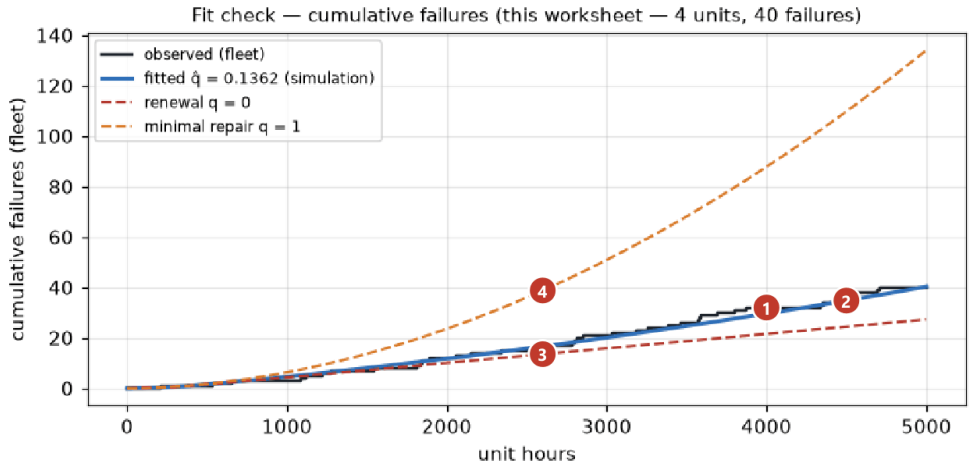
- ① ReliaEn's reading: Repairs preserve about 86 % of the unit's health (q = 0 would be full renewal, q = 1 no repair at all). The 95 % profile interval for q is [0.015, 0.593] (width 0.578); q = 0 lies outside it and q = 1 lies outside it.



Interactive Plot

Profile log-likelihood ℓ(q) (this worksheet — 4 units, 40 failures)

- ① profile ℓ(q): the line through the eleven values of the table below, so its top falls at q = 0.1; the shaded band is the interval.
- ② cutoff ℓ_max − $\chi^2_1/2$ = -286.2690: q values with the line above it are inside the interval.
- ③ q̂ = 0.1362: the true peak, between two of the eleven values.



Interactive Plot

Fit check — cumulative failures (this worksheet — 4 units, 40 failures)

- ① observed (fleet): the failures on record, stepping up to 40.
- ② fitted q̂ = 0.1362 (simulation): the fitted model, on top of the record.
- ③ renewal q = 0: the same β̂ and η̂ with full renewal, too low.
- ④ minimal repair q = 1: the same β̂ and η̂ with minimal repair, far too high.

PROFILE LOG-LIKELIHOOD ℓ(Q) — THE ELEVEN VALUES (THIS WORKSHEET — 4 UNITS, 40 FAILURES)

1. cutoff ℓ_max − $\chi^2_1(95\ \%)/2$ = -286.2690; 95 % profile interval [0.015, 0.593] (width 0.578); $\hat{q}$ = 0.1362

2 q	3 ℓ(q)	4 inside the interval?
0.0	-287.0158	no
0.1	-284.4664	yes
0.2	-284.5627	yes
0.3	-285.1661	yes
0.4	-285.6657	yes
0.5	-286.0247	yes
0.6	-286.2845	no
0.7	-286.4786	no
0.8	-286.6282	no
0.9	-286.7470	no
1.0	-286.8435	no

Profile log-likelihood ℓ(q) — the eleven values (this worksheet — 4 units, 40 failures)

- ① cutoff ℓ_max − $\chi^2_1(95\ \%)/2$ = -286.2690; 95 % profile interval [0.015, 0.593] (width 0.578); $\hat{q}$ = 0.1362.
- ② to ④ The columns q, ℓ(q), inside the interval?: from -287.0158 at q = 0.0 through -284.4664 at 0.1 to -286.8435 at 1.0; yes at 0.1 to 0.5, no elsewhere.

BAYESIAN Q — UNIFORM ON [0, 1] PRIOR

1. Profile-likelihood posterior on 101 grid points: β, η at their conditional maxima at each q, not integrated out.

Quantity	Value
2 Posterior mean	0.3038
3 Posterior median	0.2246
4 Posterior mode (grid)	0.14 (ℓ = -284.3493)
5 95 % credible interval	[0.0410, 0.9085]

Bayesian q — uniform on [0, 1] prior

- ① Profile-likelihood posterior on 101 grid points: β, η at their conditional maxima at each q, not integrated out.
- ② Posterior mean 0.3038.
- ③ Posterior median 0.2246.
- ④ Posterior mode (grid) 0.14 (ℓ = -284.3493).
- ⑤ 95 % credible interval [0.0410, 0.9085].

FIT CHECK — EXPECTED CUMULATIVE FAILURES AT THE OBSERVATION END (THIS WORKSHEET — 4 UNITS, 40 FAILURES)

1 the fitted arm by simulation; each unit is followed to its own observation end

2 Model	3 Expected total	4 Observed
5 fitted (q̂ = 0.1362)	40.28 (SE 0.11)	40
6 renewal (q = 0)	27.29	40
7 minimal repair (q = 1)	134.04	40

8 seed 20260825, N = 2,000 per unit

Fit check — expected cumulative failures at the observation end (this worksheet — 4 units, 40 failures)

- ① the fitted arm by simulation; each unit is followed to its own observation end.
- ② to ④ The columns Model, Expected total, Observed.
- ⑤ fitted (q̂ = 0.1362): 40.28 (SE 0.11) against 40.
- ⑥ renewal (q = 0): 27.29 against 40.
- ⑦ minimal repair (q = 1): 134.04 against 40.
- ⑧ seed 20260825, N = 2,000 per unit: a repeat run prints the same digits.

FORWARD CARD — 4 UNIT(S) OVER 8,760 H (THIS FIT)

Quantity	Value
1 Triple	β 1.8975, η 785.56 h, q 0.1362 (this fit)
2 Units	4
3 Horizon	8,760 h
4 Expected failures per unit	22.93 (SE 0.09) — by simulation
5 Expected failures, all units	91.71

6 seed 20260825, N = 2,000 per unit

Forward card — 4 unit(s) over 8,760 h (this fit)

- ① Triple β 1.8975, η 785.56 h, q 0.1362 (this fit).
- ② Units 4.
- ③ Horizon 8,760 h.
- ④ Expected failures per unit 22.93 (SE 0.09) — by simulation, for a unit starting new.
- ⑤ Expected failures, all units 91.71.
- ⑥ seed 20260825, N = 2,000 per unit.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence this report was made under.

Part 5 — How to interpret the results

How good are the repairs? q̂ is 0.1362: a repair leaves about 14 % of the age the pump gained since the last failure, and takes the rest away. The 95 % profile interval runs from 0.015 to 0.593, wide but clear of both ends. The tests say the same: against full renewal LR = 5.335, p = 0.0209; against minimal repair LR = 4.990, p = 0.0255. A repair is neither.

How do the pumps age? $\hat{\beta}$ = 1.8975 with $\hat{\eta}$ = 785.56 hours: above 1, so the hazard rises with virtual age. Just after its last repair each pump has a virtual age; the mean of the four is 629.21 hours, and a pump of that age runs a mean 358.70 hours to its next failure. The running time since the last repair is not counted in that age.

How far to trust q. The profile falls by only 2.4953 from its peak at $\hat{q}$ to q = 1 and by 2.6676 to q = 0, so 40 failures pin q down loosely. The Bayesian card gives one range for it: mean 0.3038, median 0.2246, 95 % credible interval 0.0410 to 0.9085. It is wider than the profile interval, because it counts the long flat side of the profile toward 1.

Does the fit match the record? With its own $\hat{\beta}$ and $\hat{\eta}$, the fitted model expects 40.28 failures at the observation end, against the 40 on record. The same $\hat{\beta}$ and $\hat{\eta}$ with q = 0 expect 27.29, and with q = 1 134.04: neither extreme comes near.

What to decide. Model the pumps with $\hat{q}$ 0.1362, not as new after repair and not as bad as old. A pump starting new is expected to fail 22.93 times in the next 8,760 hours, and four new pumps 91.71 times. Pumps already in service carry their virtual age, which this card does not include: read 91.71 as the figure for a new set, not as the forecast for these four pumps.