Smallest extreme value — weakest-link life on the data axis

Menu: Life Data ▸ Time Dependence ▸ Smallest extreme value…

Part 1 — What the function does

What it calculates. The smallest extreme value (SEV) model, also called Gumbel-min, describes the life of a weakest link measured directly on the time axis: F(t) = 1 − exp(−e^((t − μ)/σ)). Its hazard rises exponentially with age - faster than any Weibull - so it suits units that last well and then fail in a short, sharp wear-out. ReliaEn fits the location μ (the age by which 63.2% have failed) and the scale σ (the spread, in the data's own unit) by maximum likelihood, with bounds, from exact failure ages or from counts found at inspections. It gives the MTTF, the median, the B-lives with bounds, and two plots.

What ReliaEn prints	What it tells you
μ̂ and **σ̂** with bounds	Where the failures centre and how widely they spread, in hours
MTTF, **Median life**	Two centres; the MTTF lies below the median because the left tail is long
B-life and the **percentile table** with bounds	By what age a share of units has failed
Probability plot	Whether the points lie on the SEV line
R(t) and hazard	How reliability falls and how fast the hazard climbs

From which data. Either exact failure ages in one column (survivor ages, if any, in another), or - as in this chapter's example - inspection intervals: a start and an end time per row, the number of units found failed in that interval, and the number taken out still running at its end.

When an engineer uses it. When units wear out sharply late in life and the question is when they fail - and the ages involved sit far enough from zero that the model's left tail does no harm.

When not to use it.

Situation	Use instead
A life cannot be below zero, and the fit puts a share of units below age 0	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution… (the SEV on ln t)
The failure ages spread evenly on both sides of the centre	Life Data ▸ Time Dependence ▸ Logistic distribution…
The failure rate does not change with age	Life Data ▸ Time Dependence ▸ Exponential distribution…

Part 2 — The example's input data

Setting. A test rig runs 14 rubber seals and inspects them at 100, 150, 200 and 300 hours. At each inspection the technician counts the seals found failed since the last one; two seals are taken out still running along the way, at 100 and at 200 hours, and two are still running when the test stops at 300 hours. The reliability engineer wants the seals' life and an early-failure figure for planning.

Input	Value
Inspection intervals (4)	0–100, 100–150, 150–200, 200–300 hours
Failed units per interval	3, 2, 2, 3 (10 in all)
Censored units per interval	1, 0, 1, 2 (4 in all), each taken out at the interval's end
File in the worked-example zip	2-Smallest-Extreme-Value/Example-1.csv — four columns, headings Start time, End time, Failed units, Censored units, 4 rows
Confidence % / B-life level %	95 / 10 — the values the form opens with

	P	Q	R	S	T	① U	② V	③ W	④ X	Y
1						0	100	3	1	
2						100	150	2	0	
3						150	200	2	1	
4						200	300	3	2	
5										

The example data in the worksheet

- ① Column U: the start of each inspection interval, 0 to 200.
- ② Column V: the end of each interval, 100 to 300.
- ③ Column W: the seals found failed in each interval.
- ④ Column X: the seals taken out still running at each interval's end.

Part 3 — How to use it, step by step

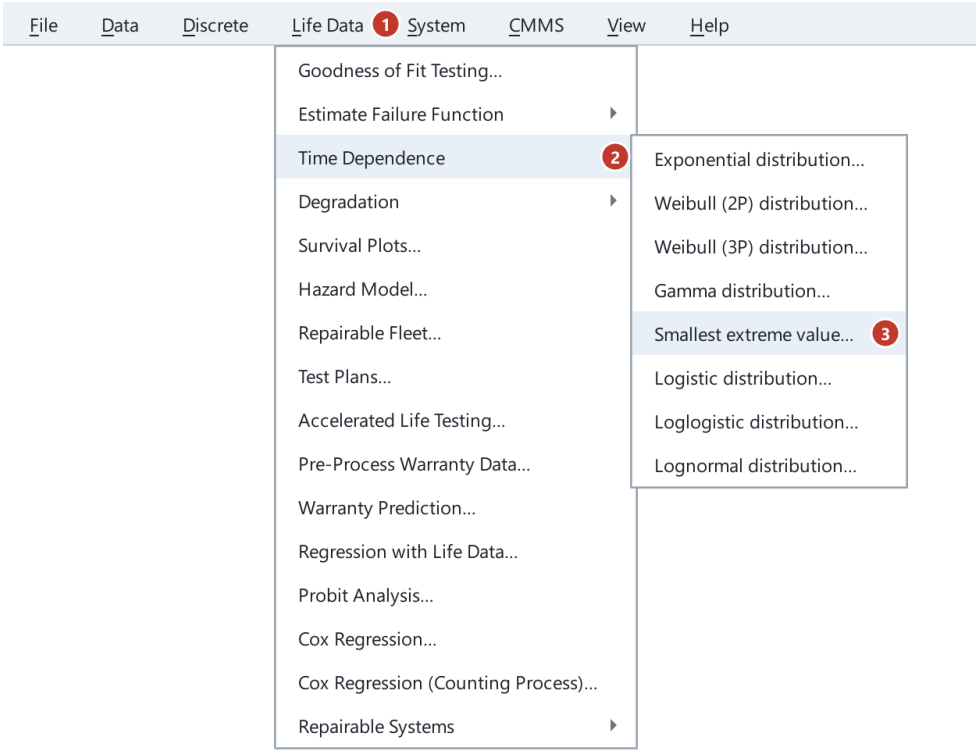
1. **Put the four columns in U to X.** Copy the 4 rows of Example-1.csv that stand under its headings Start time, End time, Failed units and Censored units into columns U to X of the worksheet, starting at row 1. Any four columns work; this example uses U to X.

	P	Q	R	S	T	① U	② V	③ W	④ X	Y
1						0	100	3	1	
2						100	150	2	0	
3						150	200	2	1	
4						200	300	3	2	
5										

Step 1 — the inspection intervals in columns U to X

- ① Column U: the interval starts.
- ② Column V: the interval ends.
- ③ Column W: the failed counts.
- ④ Column X: the censored counts.

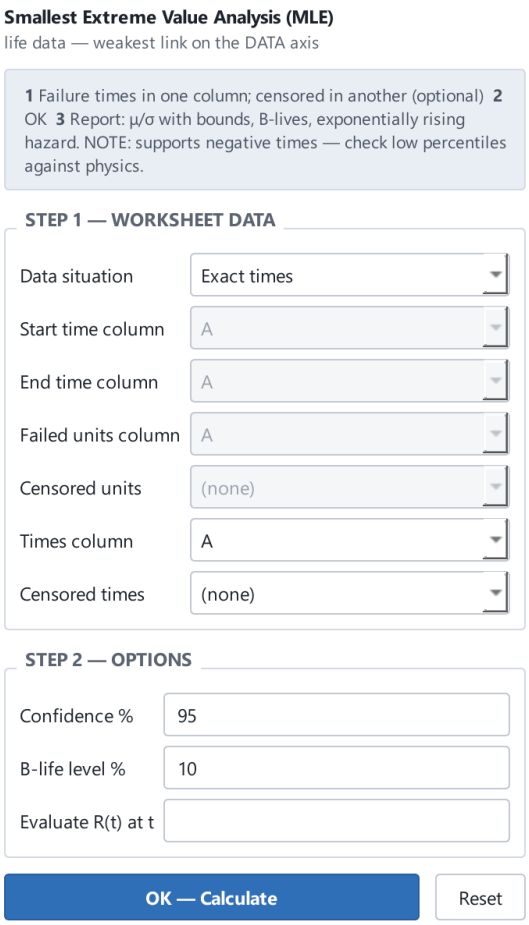
2. **Choose Life Data ▸ Time Dependence ▸ Smallest extreme value...** in the menu bar. The form Smallest Extreme Value Analysis (MLE) opens in the left panel.



Step 2 — the menu path

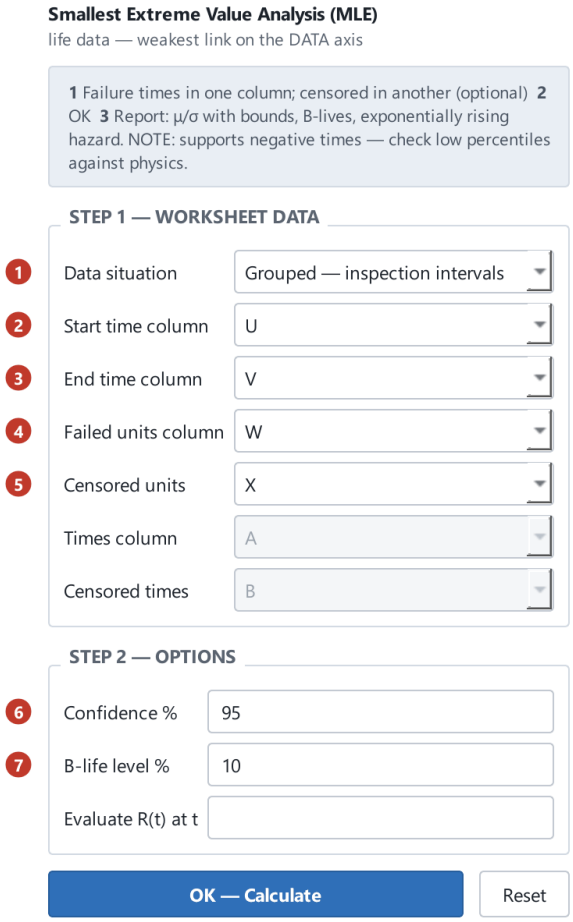
- ① Life Data: the menu.
- ② Time Dependence: the submenu.
- ③ Smallest extreme value...: the entry to click.

The form as it opens, before any box is set:



Step 2 — the form as it opens

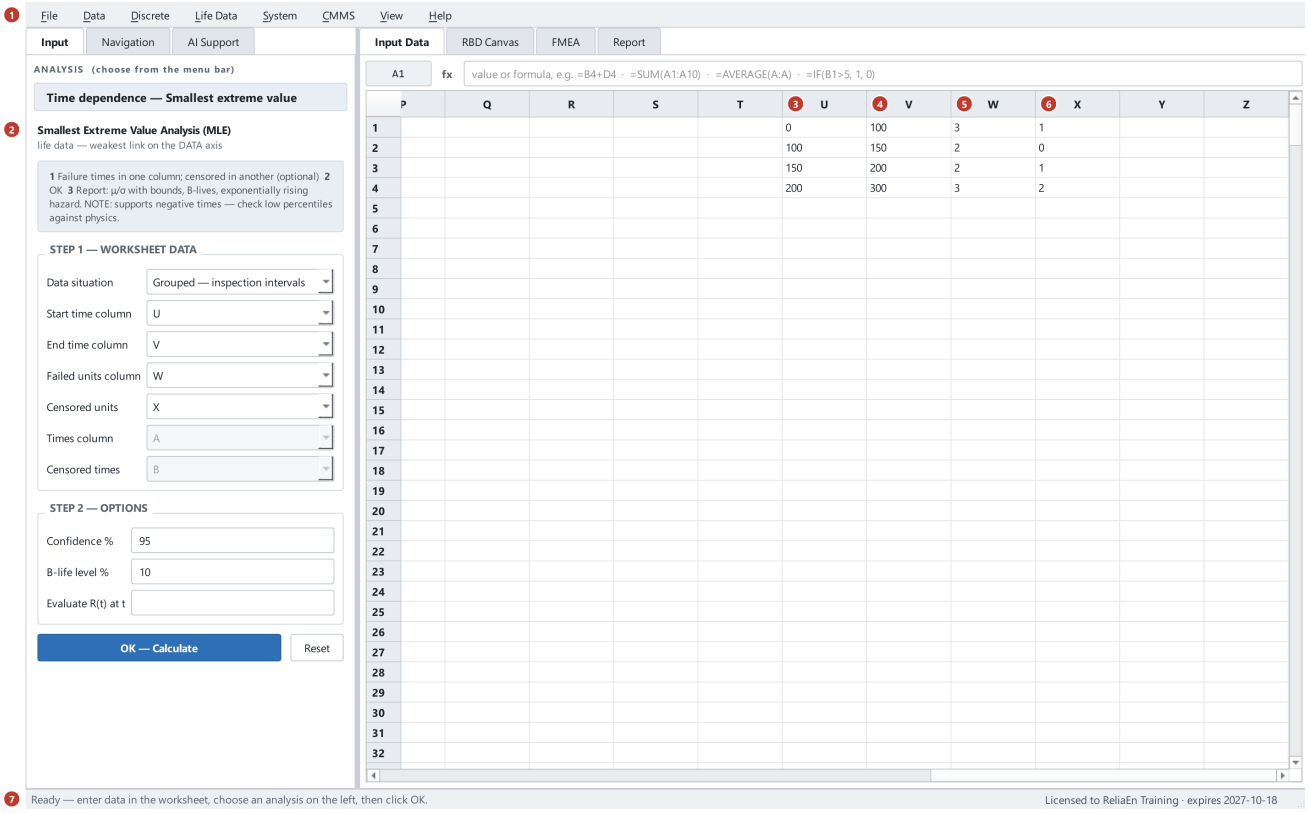
Steps 3 to 9 set or check the form's seven boxes, marked ① to ⑦ in the picture:



Steps 3 to 9 — the form set for this example

3. ① **In Data situation, choose Grouped — inspection intervals.** The four interval boxes open, and Times column and Censored times grey out.
4. ② **In Start time column, choose U.**
5. ③ **In End time column, choose V.**
6. ④ **In Failed units column, choose W.**
7. ⑤ **In Censored units, choose X.** Column X holds the seals taken out still running.
8. ⑥ **Check that Confidence % reads 95.**
9. ⑦ **Check that B-life level % reads 10.** It sets the B-life in the results. Evaluate R(t) at t stays empty.

10. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates and shows the report.



Step 10 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 9; OK — Calculate at its foot.
- ③ to ⑥ Columns U to X: the four columns of the example.
- ⑦ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — Smallest Extreme Value Analysis (MLE)

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F(t) = 1 − exp(−e^{(t−μ)/σ})

The report's head

- The title Life Data — Smallest Extreme Value Analysis (MLE), the date and time of the run, and the model in one line: F(t) = 1 − exp(−e^{(t−μ)/σ}).

	INPUT DATA	
1	n — units on test	14
2	r — failures	10
3	censored (survivors)	4
4	Data situation	grouped — inspection intervals (D78)
5	Estimation	MLE (censored location-scale)

Input data

- ① n — units on test 14.
- ② r — failures 10: the seals found failed at the four inspections.
- ③ censored (survivors) 4: the seals taken out or still running.
- ④ Data situation grouped — inspection intervals (D78): each failure is known only to lie inside its interval.
- ⑤ Estimation MLE (censored location-scale).

	RESULTS	
1	$\hat{\mu}$ — location	241.8675 (SE 29.3568) [184.3292, 299.4057]
2	$\hat{\sigma}$ — scale	90.9999 (SE 23.4436) [54.9227, 150.7751] (log-scale CI)
3	MTTF	189.34
4	Median life	208.51
5	B10 life	37.08 [-81.92, 156.09]

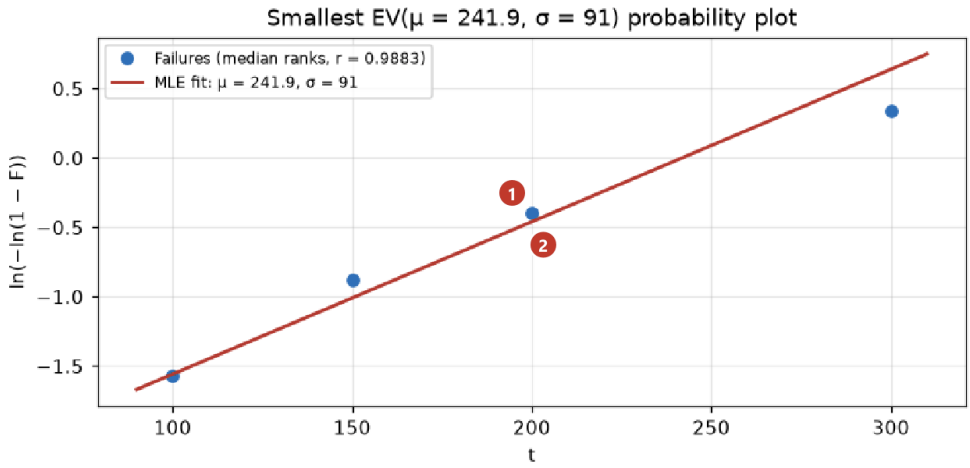
Results

- ① $\hat{\mu}$ — location 241.8675 (SE 29.3568) [184.3292, 299.4057]: by 241.9 hours, 63.2% of the seals have failed.
- ② $\hat{\sigma}$ — scale 90.9999 (SE 23.4436) [54.9227, 150.7751] (log-scale CI): the spread, in hours; the bounds are taken on the log scale, so the lower one stays above zero.
- ③ MTTF 189.34: the mean life, $\mu - 0.5772 \cdot \sigma$.
- ④ Median life 208.51: half the seals have failed by this age.
- ⑤ B10 life 37.08 [-81.92, 156.09]: one seal in ten has failed by 37 hours - and the lower bound is below zero.

	INTERPRETATION
1	Smallest-extreme-value (Gumbel-min): the life of a weakest-link chain measured on the DATA axis — the hazard rises EXPONENTIALLY with age, faster than any Weibull power law. NOTE: the model supports negative times; low percentiles (and their bounds) can go below zero — if that is physically meaningless here, the Weibull (= SEV on ln t) is the better frame.

Interpretation

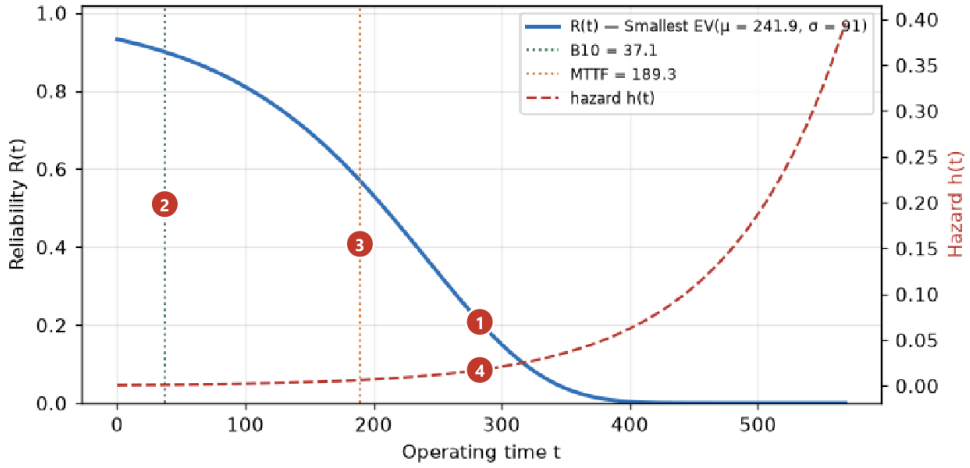
- ① ReliaEn's reading: Smallest-extreme-value (Gumbel-min): the life of a weakest-link chain measured on the DATA axis — the hazard rises EXPONENTIALLY with age, faster than any Weibull power law. NOTE: the model supports negative times; low percentiles (and their bounds) can go below zero — if that is physically meaningless here, the Weibull (= SEV on ln t) is the better frame.



Interactive Plot

Smallest EV(μ = 241.9, σ = 91) probability plot

- ① Failures (median ranks, r = 0.9883): the share failed by each inspection - at 100, 150, 200 and 300 hours - on SEV paper.
- ② MLE fit: μ = 241.9, σ = 91: the fitted line.



Interactive Plot

Reliability and hazard

- ① R(t) — Smallest EV(μ = 241.9, σ = 91): the share of seals still running at each age. It starts at about 0.93 at age 0, not at 1.
- ② B10 = 37.1: where R crosses 90%.
- ③ MTTF = 189.3: the mean life; R there is about 0.57.
- ④ hazard h(t): nearly flat for the first 200 hours, then climbing steeply - the SEV signature.

PERCENTILE TABLE — B-LIVES WITH BOUNDS

1 B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.

2 Percentile	3 B-life	4 95% bounds
1%	-176.7	[-396.5, 43.06]
5%	-28.42	[-177.2, 120.4]
5 10%	37.08	[-81.92, 156.1]
25%	128.5	[46.78, 210.2]
50%	208.5	[148.3, 268.7]
75%	271.6	[212.3, 330.9]
90%	317.8	[249.1, 386.5]
95%	341.7	[265.7, 417.7]
99%	380.8	[290.7, 471]

Percentile table — B-lives with bounds

- ① B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.
- ② to ④ The columns Percentile, B-life, 95% bounds.
- ⑤ The 10% row: 37.08 [-81.92, 156.1] - the B10 of the results card. The 1% and 5% rows above it are below zero: -176.7 and -28.42.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

Part 5 — How to interpret the results

When the seals fail. Half the seals have failed by 208.51 hours (the median), and 63.2% by 241.8675 ($\hat{\mu}$); the spread $\hat{\sigma}$ is 90.9999 hours. The mean, 189.34, sits below the median: the SEV's long tail is on the left, towards early failures. The four inspection points lie close to the fitted line (r = 0.9883), so the shape fits the middle and late life well.

Where the model stops making sense. The early percentiles go below zero - B1 is -176.7 and B5 is -28.42 - and the R(t) curve starts at about 0.93: the fitted model counts about 7% of the seals as failed before the test began. B10 is 37.08, but its lower bound -81.92 means the data cannot defend any positive B10. Fourteen seals counted at four inspections say little about the early tail - the first three failures are known only to lie somewhere before 100 hours.

What to do with it. Use the SEV fit for the median and the late life. For an early-failure figure to plan with, fit the same intervals with the Weibull (2P) distribution, which cannot go below zero - as ReliaEn's own reading advises.