Weibull 3P — the failure-free life γ, tested before it is used

Menu: Life Data ▸ Time Dependence ▸ Weibull (3P) distribution...

Part 1 — What the function does

What it calculates. Some populations cannot fail before they have run for a certain time: a coating must wear through, a crack must grow, a lubricant film must break down. The three-parameter Weibull model adds that **failure-free life γ** to the shape β and scale η: no unit fails before γ, and after γ the population ages as an ordinary Weibull. ReliaEn fits both the 3-parameter model and the 2-parameter model (γ = 0), tests the difference with a likelihood ratio (LR) test, and reports ONE model - you do not choose. Every result (MTTF, median, B-lives, service intervals) comes from the model reported.

What ReliaEn prints	What it tells you
γ̂ with its **profile interval**	How long the failure-free life is, and how precisely the data locate it
β̂ and **η̂** with bounds	The shape and scale after γ
LR test and **Model reported**	Whether a failure-free life is supported at your confidence level, and which model the results use
MTTF, **median**, **B10**, **SERVICE INTERVALS**, **percentile table**	The life figures of the reported model
Curvature router and **method comparison**	Whether a lognormal explains the curve as well, and where rank regression would put γ - printed for checking, never used for the result

From which data. The failure times in one worksheet column, one row per failed unit, at least three; censored (suspended) ages, if any, in a different column. Every age measured from the same, correct starting time.

When an engineer uses it. When a Weibull (2P) plot curves downward at the left, or when the physics says no failure can happen before some age, and the question is how long that age is.

When not to use it.

Situation	Use instead
The Weibull (2P) plot is straight	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution...
The curve comes from growth that multiplies, not from a threshold	Life Data ▸ Time Dependence ▸ Lognormal distribution... - the curvature router tells you
You only need to check which family fits	Life Data ▸ Goodness of Fit Testing...
Fewer than three failures	No three-parameter fit is possible; the form refuses

Part 2 — The example's input data

Setting. Sixty gearbox bearings, all run to failure. The physics says no bearing fails before its lubricant film breaks down; the reliability engineer wants to know how long that takes, and whether the data support it at all.

Input	Value
Failure ages, operating hours (60)	72.4, 76.4, 79.7, … (the sixty values of the file)
File in the worked-example zip	2-Weibull-3P/Example-1.csv — one column, heading Times, 60 rows
Confidence %	95 — the value the form opens with
B-life level %	10 — the value the form opens with

	① A	B
1	72.4	
2	76.4	
3	79.7	
4	97.3	
5	100.8	
6	123.1	
7	123.6	
8	126.4	
9	126.4	
10	134.5	
11	141.7	
12	144.7	
13	147.0	
14	148.1	
15	149.0	
16	161.0	
17	169.0	
18	179.1	
19	180.1	
20	180.3	
21	181.9	
22	186.2	
23	188.2	
24	188.5	
25	190.5	
26	197.8	
27	201.1	
28	212.3	
29	214.3	
30	214.4	
31	214.5	
32	215.6	

The example data in the worksheet

- ① Column A: the 60 failure ages, one per row, in operating hours.

Part 3 — How to use it, step by step

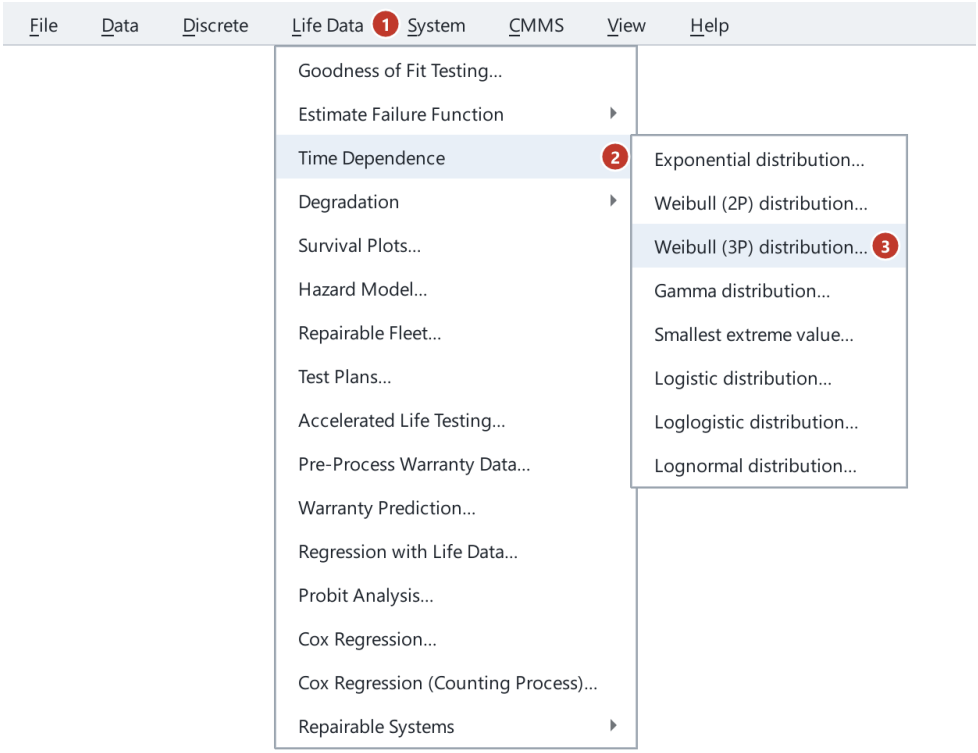
1. **Put the failure ages in column A.** Copy the 60 values of Example-1.csv that stand under its heading Times into column A of the worksheet, starting at row 1.

	① A	B
1	72.4	
2	76.4	
3	79.7	
4	97.3	
5	100.8	
6	123.1	
7	123.6	
8	126.4	
9	126.4	
10	134.5	
11	141.7	
12	144.7	
13	147.0	
14	148.1	
15	149.0	
16	161.0	
17	169.0	
18	179.1	
19	180.1	
20	180.3	
21	181.9	
22	186.2	
23	188.2	
24	188.5	
25	190.5	
26	197.8	
27	201.1	
28	212.3	
29	214.3	
30	214.4	
31	214.5	
32	215.6	

Step 1 — the failure ages in column A

- ① Column A: the 60 failure ages, starting 72.4, 76.4, 79.7.

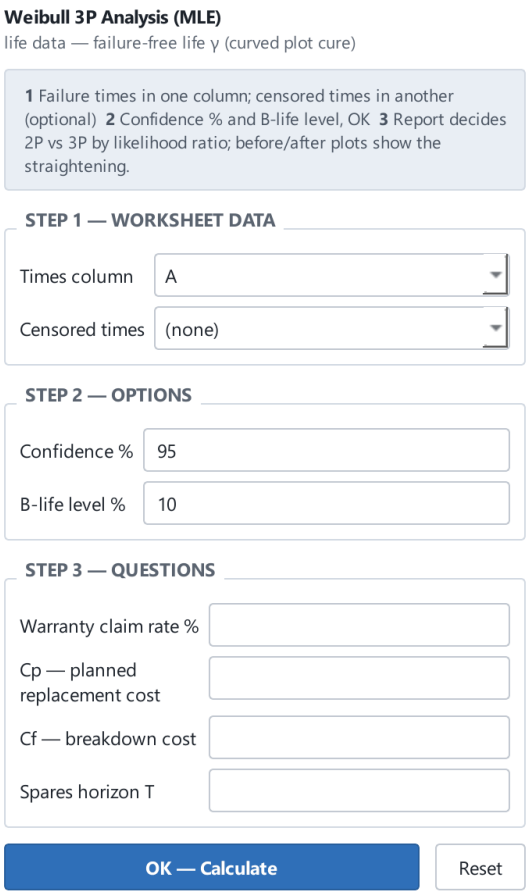
2. **Choose Life Data ▸ Time Dependence ▸ Weibull (3P) distribution...** in the menu bar. The form Weibull 3P Analysis (MLE) opens in the left panel.



Step 2 — the menu path

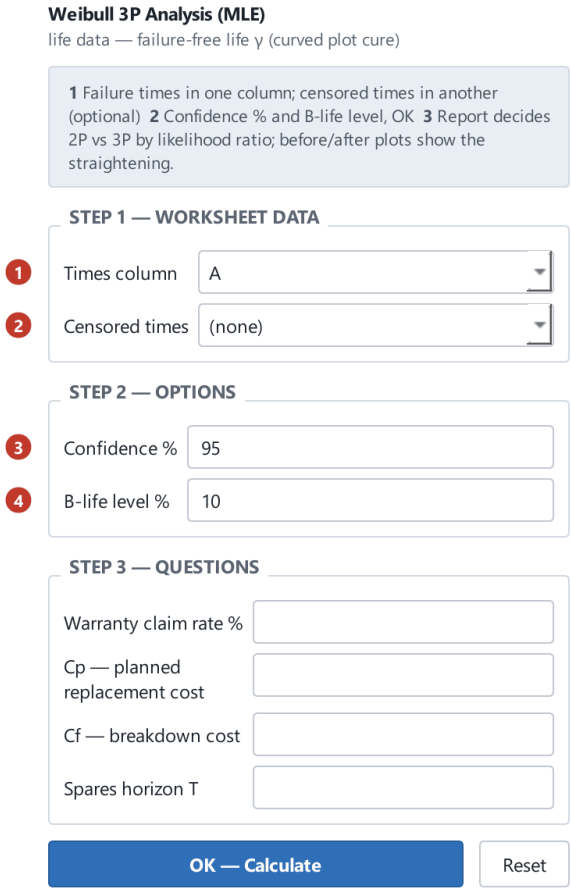
- ① Life Data: the menu.
- ② Time Dependence: the submenu.
- ③ Weibull (3P) distribution...: the entry to click.

The form as it opens, before any box is set:



Step 2 — the form as it opens

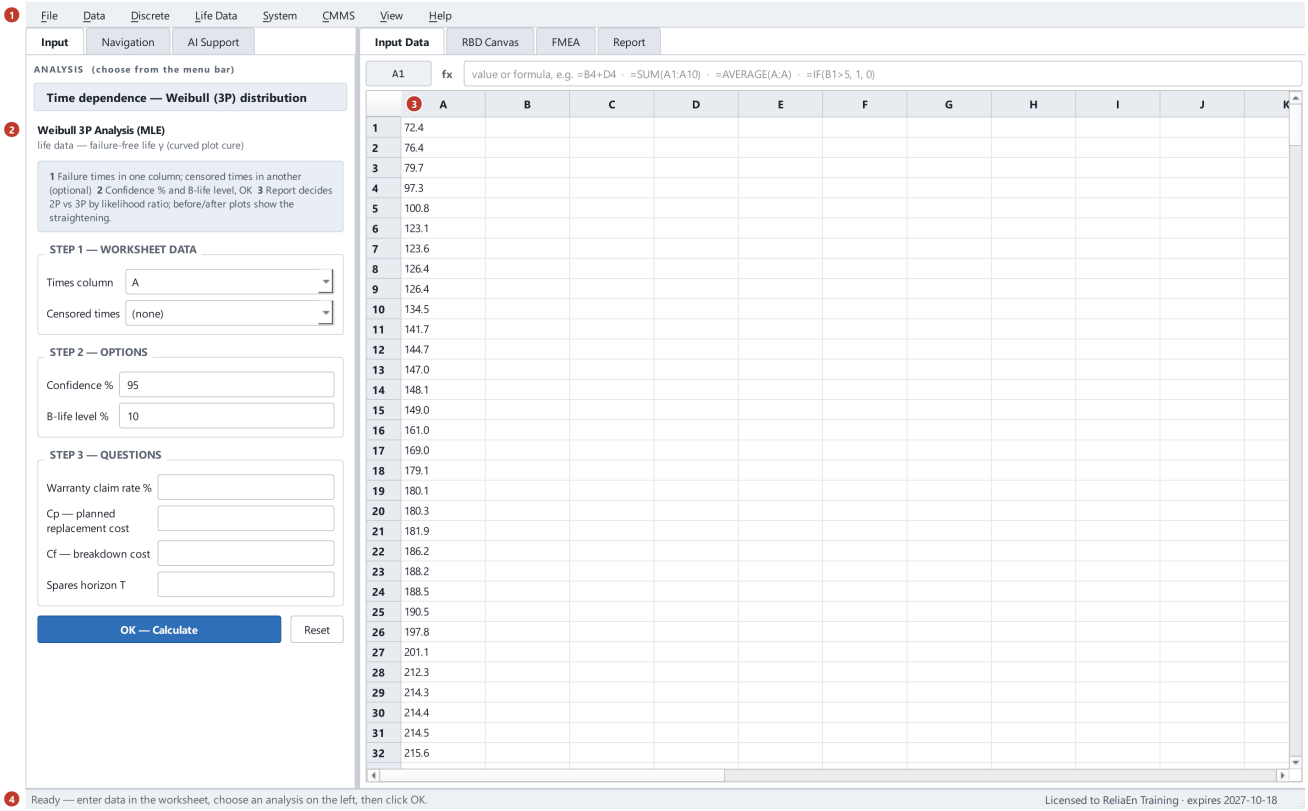
Steps 3 to 6 set the form's four boxes, marked ① to ④ in the picture:



Steps 3 to 6 — the form set for this example

3. ① **In Times column, choose A.** The column that holds the failure ages.
4. ② **Leave Censored times at (none).** Every bearing was run to failure.
5. ③ **Check that Confidence % reads 95.** It sets the bounds, the profile interval of γ and the level of the LR test.
6. ④ **Check that B-life level % reads 10.** The B-life printed in the results. The optional questions under STEP 3 — QUESTIONS stay empty in this example.

7. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates and shows the report.



Step 7 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 6; OK — Calculate at its foot.
- ③ Column A: the 60 failure ages.
- ④ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — Weibull (3P) — Time Dependence

2026-10-30 18:01:55

$R(t) = e^{-((t-\gamma)/\eta)^{\beta}}$ for $t > \gamma$; $B_q = \gamma + \eta(-\ln(1-q))^{1/\beta}$

The report's head

- The title Life Data — Weibull (3P) — Time Dependence, the date and time of the run, and the model in one line: R(t) = e^(−((t−γ)/η)^β) for t > γ; B_q = γ + η(−ln(1−q))^(1/β).

	INPUT DATA	
1	n — units on test	60
2	r — failures	60
3	censored (survivors)	0
4	Estimation	γ-profile MLE (via the verified 2P engine)

Input data

- ① n — units on test 60.
- ② r — failures 60. Check r first: a blank cell is skipped.
- ③ censored (survivors) 0.
- ④ Estimation γ-profile MLE (via the verified 2P engine): for each candidate γ the ages are shifted to t − γ and the 2-parameter model is fitted by maximum likelihood; γ̂ is the γ with the highest likelihood.

	RESULTS	
1	β̂ — shape (conditional on γ̂)	1.74075 [1.43258, 2.11522]
2	η̂ — scale (conditional on γ̂)	185.153 [158.909, 215.732]
3	γ̂ — failure-free life (profile interval)	62.2904 [15.8277, 72.2646]
4	MTTF	227.24
5	Median life	212.29
6	B10 life (conditional on γ̂)	113.12 [98.74, 133.17]

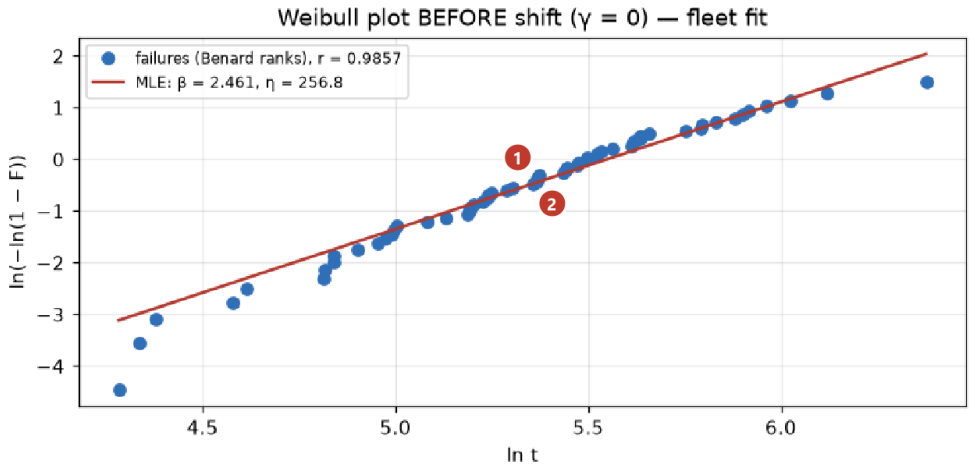
Results

- ① β̂ — shape (conditional on γ̂) 1.74075 [1.43258, 2.11522]: the shape after the failure-free life; above 1, wear-out.
- ② η̂ — scale (conditional on γ̂) 185.153 [158.909, 215.732].
- ③ γ̂ — failure-free life (profile interval) 62.2904 [15.8277, 72.2646]: no bearing is expected to fail before about 62 hours; the interval is wide - sixty failures locate a threshold only roughly.
- ④ MTTF 227.24: the mean life, failure-free life included.
- ⑤ Median life 212.29: half the bearings fail by this age.
- ⑥ B10 life (conditional on γ̂) 113.12 [98.74, 133.17]: one bearing in ten has failed by this age.

	INTERPRETATION
1	FAILURE-FREE LIFE CONFIRMED: no failure is possible before γ̂ = 62.3 — the population needs that long to accumulate enough damage to fail at all (incubation, break-in reserve). After γ the fleet behaves as a Weibull with β = 1.741. MTTF = 227.24; 10% of units fail by B10 = 113.12.

Interpretation

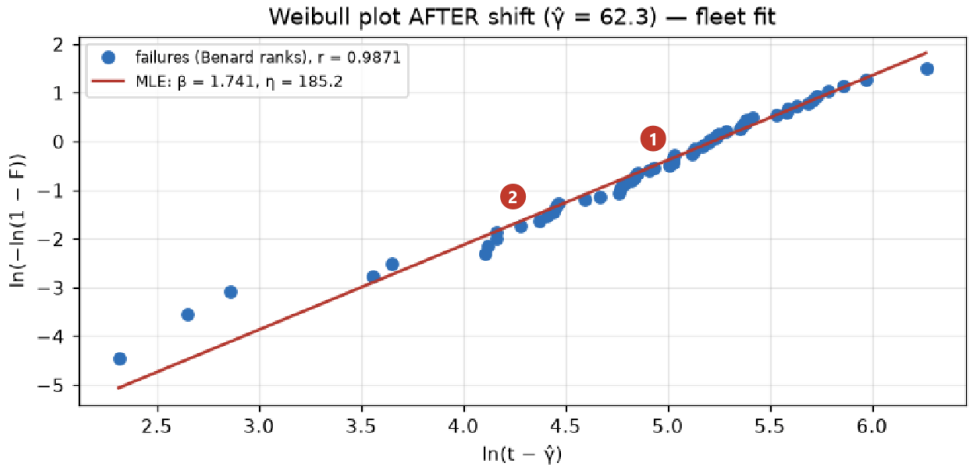
- ① ReliaEn's verdict: FAILURE-FREE LIFE CONFIRMED: no failure is possible before γ̂ = 62.3 — the population needs that long to accumulate enough damage to fail at all (incubation, break-in reserve). After γ the fleet behaves as a Weibull with β = 1.741. MTTF = 227.24; 10% of units fail by B10 = 113.12.



Interactive Plot

The Weibull plot before the shift

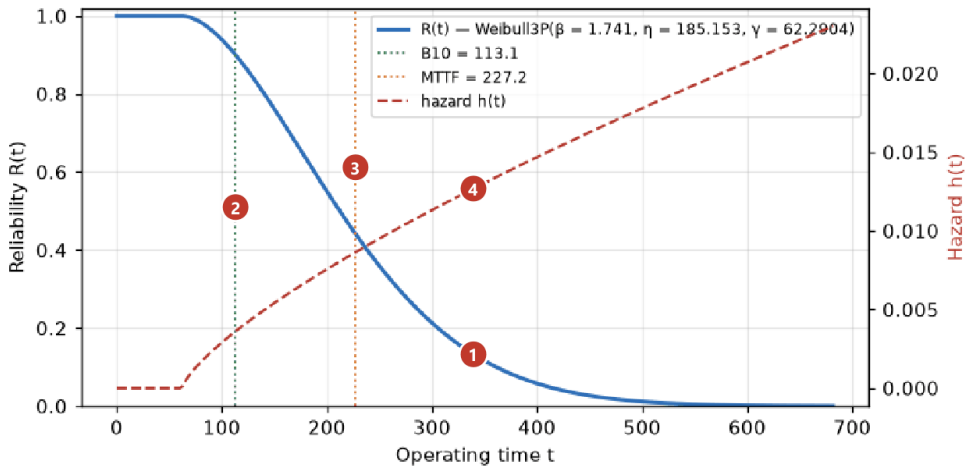
- ① failures (Benard ranks), r = 0.9857: the sixty ages on the 2-parameter Weibull plot (γ = 0); they curve downward at the left.
- ② MLE: β = 2.461, η = 256.8: the 2-parameter fit, which misses the curve at both ends.



Interactive Plot

The Weibull plot after the shift

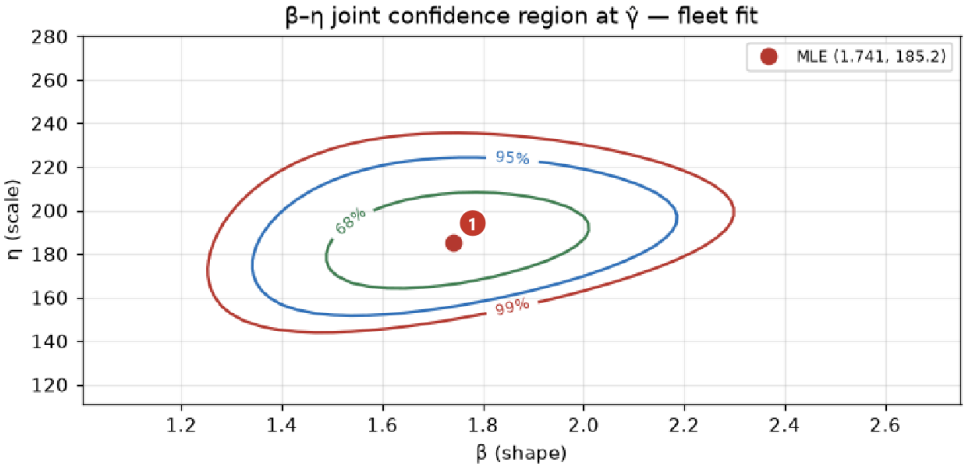
- ① failures (Benard ranks), r = 0.9871: the same ages shifted by γ̂; the curve straightens.
- ② MLE: β = 1.741, η = 185.2: the 3-parameter fit after the shift.



Interactive Plot

Reliability and hazard

- ① R(t) — Weibull3P(β = 1.741, η = 185.153, γ = 62.2904): exactly 1 up to γ̂, then falling.
- ② B10 = 113.1: where R crosses 90%.
- ③ MTTF = 227.2: the mean life on the curve.
- ④ hazard h(t): zero before γ̂, then rising - wear-out.



Interactive Plot

The joint confidence region of β and η

- ① MLE (1.741, 185.2): the estimate, inside its 68%, 95% and 99% regions, with γ held at γ̂.

FAILURE-FREE LIFE — DOES Γ EXIST? (LR TEST, FLEET FIT)

1 p = 0.0212 < 0.05 — a failure-free life is CONFIRMED: γ̂ = 62.3. All results above use the 3P model; the plots below show the curvature before the shift and the straightening after it.

Quantity	Value
2 γ̂ — free failure-free life	62.29 [15.8, 72.3]
3 3P fit	β = 1.7408, η = 185.15, lnL = -354.7593
4 2P fit (γ = 0)	β = 2.4610, η = 256.81, lnL = -357.4139
5 LR = 2(lnL₃ − lnL₂)	5.3092
6 p-value (χ², 1 dof)	0.0212
7 Model reported	Weibull 3P (failure-free life)

8 Nested models; the boundary hypothesis makes χ²(1) approximate — stated, not hidden. γ's interval is PROFILE-likelihood (a threshold parameter has no trustworthy Fisher bounds).

Failure-free life — does γ exist? (LR test, fleet fit)

- ① p = 0.0212 < 0.05 — a failure-free life is CONFIRMED: γ̂ = 62.3. All results above use the 3P model; the plots below show the curvature before the shift and the straightening after it.
- ② γ̂ — free failure-free life 62.29 [15.8, 72.3].
- ③ 3P fit β = 1.7408, η = 185.15, lnL = -354.7593.
- ④ 2P fit (γ = 0) β = 2.4610, η = 256.81, lnL = -357.4139.
- ⑤ LR = 2(lnL₃ − lnL₂) 5.3092: twice the gain in log-likelihood from adding γ.
- ⑥ p-value (χ², 1 dof) 0.0212: below 0.05, so γ = 0 is rejected at 95%.
- ⑦ Model reported Weibull 3P (failure-free life).
- ⑧ Nested models; the boundary hypothesis makes χ²(1) approximate — stated, not hidden. γ's interval is PROFILE-likelihood (a threshold parameter has no trustworthy Fisher bounds).

ESTIMATION METHOD COMPARISON — FLEET FIT

1 The RR route picks γ where the plot is straightest: r rises from 0.9857 (γ = 0) to 0.9933 — shown for cross-checking against rank-regression-based software.

2 Method	3 γ	4 β	5 η	6 r
7 MLE (reported)	62.3	1.7408	185.15	—
8 Rank regression (max-correlation γ)	45.4	1.9976	206.12	0.9933

Estimation method comparison — fleet fit

- ① The RR route picks γ where the plot is straightest: r rises from 0.9857 (γ = 0) to 0.9933 — shown for cross-checking against rank-regression-based software.
- ② to ⑥ The columns Method, γ, β, η, r.
- ⑦ MLE (reported): γ 62.3, β 1.7408, η 185.15 - the fit the results use.
- ⑧ Rank regression (max-correlation γ): γ 45.4, β 1.9976, η 206.12, r 0.9933 - a different route, printed for comparison only.

CURVATURE ROUTER — Γ OR LOGNORMAL? (SAME DATA)

1 Lognormal(μ = 5.337, σ = 0.4348) explains this data best by AIC.

2 Model	3 ln-likelihood	4 AIC	
5 Weibull 2P	-357.414	722.8	
6 Weibull 3P	-354.759	721.5	
7 Lognormal(μ = 5.337, σ = 0.4348)	-355.368	718.7	◄ best

8 A curved Weibull plot has TWO common cures — a failure-free life (3P) or the lognormal — and small samples often cannot separate them; the mechanism decides (threshold physics vs multiplicative growth).

Curvature router — γ or lognormal? (same data)

- ① Lognormal(μ = 5.337, σ = 0.4348) explains this data best by AIC.
- ② to ④ The columns Model, ln-likelihood, AIC.
- ⑤ Weibull 2P: -357.414, AIC 722.8.
- ⑥ Weibull 3P: -354.759, AIC 721.5.
- ⑦ Lognormal(μ = 5.337, σ = 0.4348): -355.368, AIC 718.7, ◄ best.
- ⑧ A curved Weibull plot has TWO common cures — a failure-free life (3P) or the lognormal — and small samples often cannot separate them; the mechanism decides (threshold physics vs multiplicative growth).

BEFORE BANKING ON Γ — CHECK THE TIME ORIGIN

1 A failure-free life of 62 is statistically indistinguishable from a 62-unit BOOKKEEPING OFFSET — ages logged from installation instead of commissioning, or from the last overhaul instead of new. Verify the data's clock before treating γ as physics.

Before banking on γ — check the time origin

- ① A failure-free life of 62 is statistically indistinguishable from a 62-unit BOOKKEEPING OFFSET — ages logged from installation instead of commissioning, or from the last overhaul instead of new. Verify the data's clock before treating γ as physics.

SERVICE INTERVALS — OPERATING TIME TO STAY AT OR ABOVE TARGET RELIABILITY

1 Model: Weibull3P(β = 1.741, η = 185.153, γ = 62.2904) — service before the unit accumulates this much operating time to keep reliability at or above the target.

2 Target R	3 Operating time t
95%	95.9
4 90%	113.1
85%	127.5
80%	140.5
75%	152.8
70%	164.7
65%	176.4
60%	188.2
55%	200.1
50%	212.3
45%	225.0
40%	238.4
35%	252.7
30%	268.3
25%	285.7
20%	305.7
15%	329.8
10%	361.2
5%	410.0

SERVICE INTERVALS — operating time to stay at or above target reliability

- ① Model: Weibull3P(β = 1.741, η = 185.153, γ = 62.2904) — service before the unit accumulates this much operating time to keep reliability at or above the target.
- ② Target R: the reliability to keep, from 95% down to 5%.
- ③ Operating time t: the longest run that keeps it.
- ④ The 90% row: 113.1 hours - B10.

PERCENTILE TABLE — B-LIVES WITH BOUNDS

1 B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.

2 Percentile	3 B-life	4 95% bounds
1%	75.47	[69.66, 85.84]
5%	95.9	[84.67, 112.8]
5 10%	113.1	[98.74, 133.2]
25%	152.8	[133.7, 177]
50%	212.3	[188.8, 240.1]
75%	285.7	[255.4, 320.7]
90%	361.2	[319, 410.5]
95%	410	[357.5, 471.9]
99%	507.5	[430.1, 601.1]

Percentile table — B-lives with bounds

- ① B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.
- ② to ④ The columns Percentile, B-life, 95% bounds.
- ⑤ The 10% row: 113.1 [98.74, 133.2] - the B10 of the results card.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

Part 5 — How to interpret the results

The verdict. The LR test confirms a failure-free life at 95%: p = 0.0212 is below 0.05, and ReliaEn reports the 3-parameter model. No bearing is expected to fail before $\hat{\gamma}$ = 62.2904 hours; after that the bearings wear out with $\hat{\beta}$ = 1.74075. One bearing in ten has failed by 113.12 hours (B10), half by 212.29.

How precise the threshold is. The profile interval for γ, [15.8277, 72.2646], is wide: sixty failures locate a threshold only roughly. Plan on the lower end when a failure-free period carries a commitment - a warranty that relies on it, or an inspection that is skipped because of it.

Two warnings the page gives you. The curvature router finds the lognormal best by AIC (718.7 against 721.5 for the 3-parameter model): statistics alone cannot choose between a threshold and growth that multiplies - the failure mechanism must. And a failure-free life of 62 hours looks exactly like a 62-hour error in the ages' starting time: check that every age counts from commissioning, not from installation or the last overhaul, before treating γ as physics.

What follows for maintenance. The service-interval table gives the longest run that keeps each target reliability under the reported model - 113.1 hours for 90%. With $\hat{\beta}$ above 1 the bearings wear out, so a planned replacement can pay; the costs in the form's optional questions would let ReliaEn weigh it.