Exponential — the failure rate as it changes with age, from the fleet's own survivor curve

Menu: Life Data ▸ Time Dependence ▸ Exponential distribution…

Part 1 — What the function does

What it calculates. A fleet of like units started together at age 0, and you know the age at which each one failed (and the age of any unit removed before it failed). The function builds the fleet's survivor table, fits a quadratic curve Ns(t) through it, and reads the failure rate from that curve: λ(t) = Nf′(t)/Ns(t). No shape of the failure rate is assumed in advance. From λ(t) it gives the reliability R(t) = e^(−∫λ), the mean life, and R at a target age you choose. Two pre-tests on the raw ages - the χ^2 exponential test and the Barlow–Proschan trend test - say whether the data can prove the rate is not constant.

What ReliaEn prints	What it tells you
Survivor table and **Ns(t)**, with its **R^2**	How many units survive to each age, and how well the quadratic curve follows them
λ(t) = Nf′(t)/Ns(t), when new and at mid-life	Whether the failure rate rises, falls or stays nearly constant with age
MTBF = ∫ R(t) dt and **R at the target**	The fleet's mean life, and the share that survives to the age you name
χ^2 exponential test and **Barlow–Proschan test**	Whether the raw ages can reject a constant rate, and in which direction they point
Regression usability	Whether R^2 reaches 0.6, the course rule for using λ(t) at all

From which data. One worksheet column with the failure age of every unit, one row per failed unit; two units that failed at the same age are two rows. Optionally a second column with the ages of units removed without failing. At least 3 ages, each above 0.

When an engineer uses it. When you want to see how the failure rate of a fleet changes with age before choosing a model or a maintenance policy - for example to see whether wear-out is setting in.

When not to use it.

Situation	Use instead
You know the rate is constant and want its confidence bounds	Life Data ▸ Estimate Failure Function ▸ Complete data…, Time-censored data… or Failure-censored data…
You only want to test whether the rate is constant	Life Data ▸ Goodness of Fit Testing…
You want a fitted life distribution with bounds on its parameters	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution…
One repairable system fails again and again	Life Data ▸ Estimate Failure Function ▸ Repairable unit data…

Part 2 — The example's input data

Setting. A boiler house has ten feed pumps that started together. All ten have failed; two failed at the same age, 89 operating hours. The reliability engineer wants to know whether the pumps wear out, their mean life, and how many survive to 70 hours.

Input	Value
Failure ages, operating hours (10)	10, 27, 45, 60, 89, 89, 109, 143, 167, 220
File in the worked-example zip	2-Exponential/Example-1.csv — one column, heading Failure time, 10 rows
R at target time (optional)	70 — typed in the form
Integration step Δt	2 — the value the form opens with

	① A	B
1	10	
2	27	
3	45	
4	60	
5	89	
6	89	
7	109	
8	143	
9	167	
10	220	
11		

The example data in the worksheet

- ① Column A: the 10 failure ages, one per row; the two pumps that failed at 89 are two rows.

Part 3 — How to use it, step by step

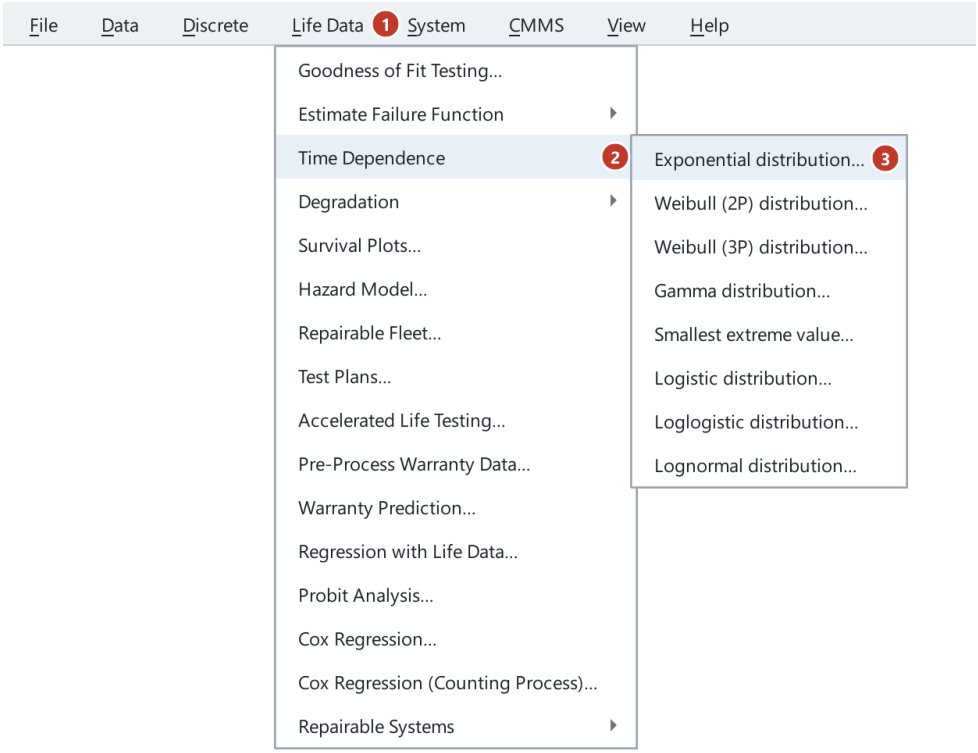
1. **Put the failure ages in column A.** Copy the 10 values of Example-1.csv that stand under its heading Failure time into column A of the worksheet, starting at row 1. Enter a shared age once per unit.

	① A	B
1	10	
2	27	
3	45	
4	60	
5	89	
6	89	
7	109	
8	143	
9	167	
10	220	
11		

Step 1 — the failure ages in column A

- ① Column A: the 10 failure ages, 10 to 220.

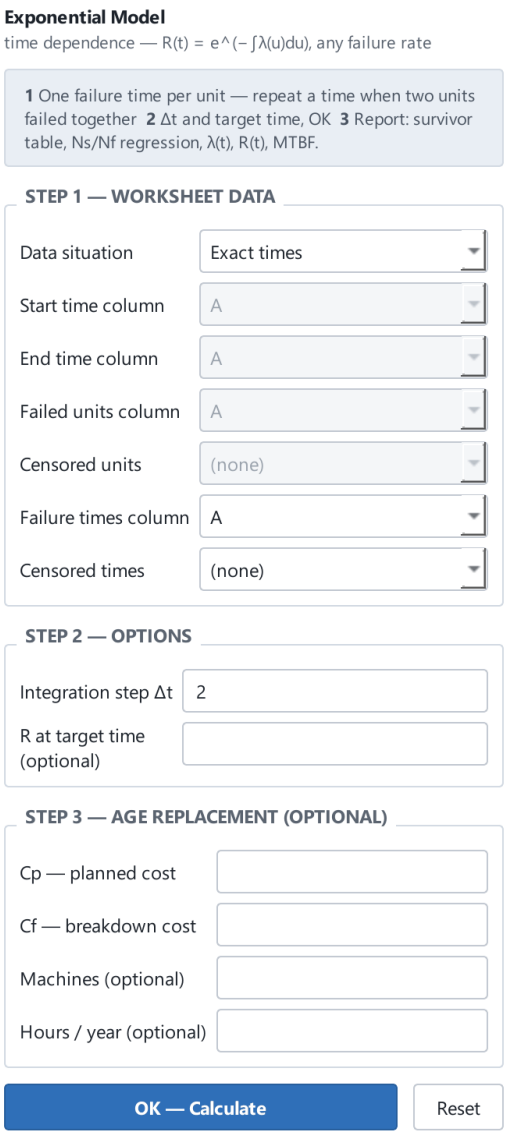
2. **Choose Life Data ▸ Time Dependence ▸ Exponential distribution...** in the menu bar. The form Exponential Model opens in the left panel.



Step 2 — the menu path

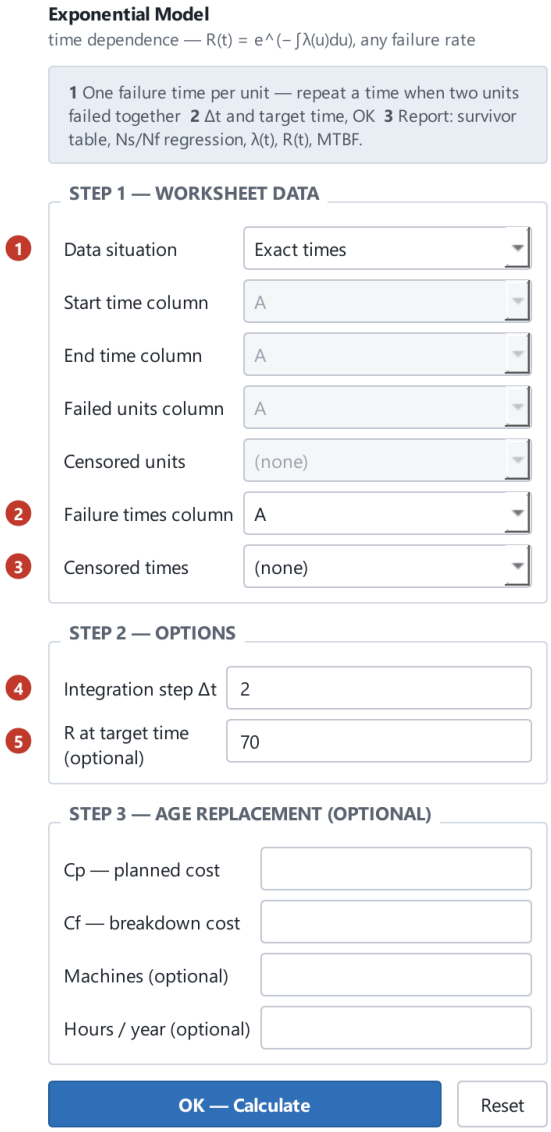
- ① Life Data: the menu.
- ② Time Dependence: the submenu.
- ③ Exponential distribution...: the entry to click.

The form as it opens, before any box is set:



Step 2 — the form as it opens

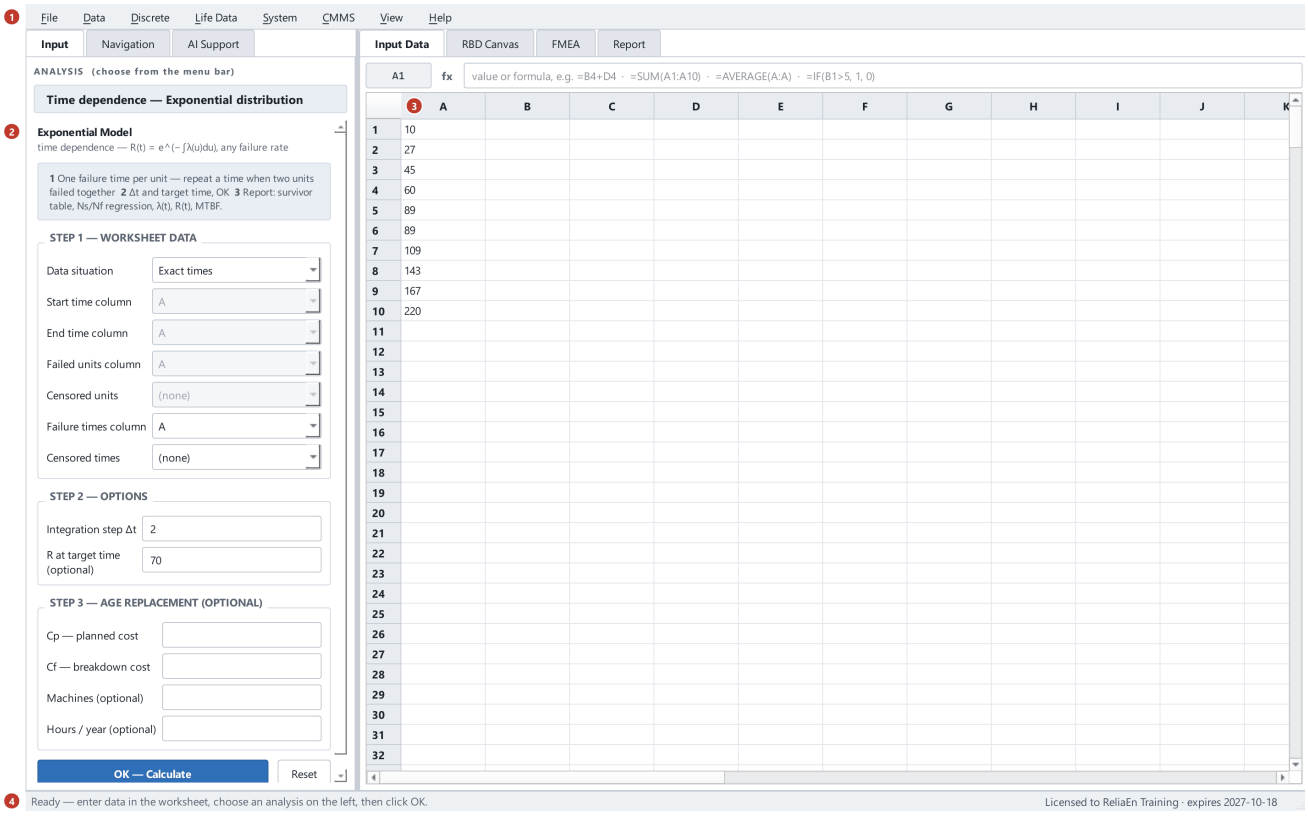
Steps 3 to 7 set the form's five boxes, marked ① to ⑤ in the picture:



Steps 3 to 7 — the form set for this example

3. ① **Check that Data situation reads Exact times.** One age per unit. The four boxes under it, from Start time column to Censored units, are for the other entry, inspection intervals, and stay greyed out.
4. ② **In Failure times column, choose A.** The column that holds the failure ages.
5. ③ **Leave Censored times at (none).** No pump was removed before it failed.
6. ④ **Check that Integration step Δt reads 2.** The step of the age grid on which ReliaEn integrates λ(t) and R(t).
7. ⑤ **In R at target time (optional), type 70.** ReliaEn reports the share of pumps that survive to 70 hours. The four boxes under STEP 3 — AGE REPLACEMENT (OPTIONAL) stay empty in this example.

8. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates and shows the report.



Step 8 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 7; OK — Calculate at its foot.
- ③ Column A: the 10 failure ages.
- ④ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — Exponential — Time Dependence

2026-10-30 15:41:46

$R(t) = e^{-\int \lambda(u)du}$; $\lambda(t) = Nf'(t)/Ns(t)$; $MTBF = \int R(t)\,dt$

The report's head

- The title Life Data — Exponential — Time Dependence, the date and time of the run, and the model in one line: R(t) = e^(−∫λ(u)du), λ(t) = Nf′(t)/Ns(t), MTBF = ∫ R(t) dt.

INPUT DATA

1	N0 — fleet size (from the events)	10
2	failures	10
3	censored (survivors)	0
4	Δt — integration step	2
5	Target time	70

Input data

- ① N0 — fleet size (from the events) 10: all the units, failures plus removals. Check N0 first: a blank cell is skipped, and N0 would then be lower than the units you entered.
- ② failures 10.
- ③ censored (survivors) 0.
- ④ Δt — integration step 2.
- ⑤ Target time 70: the age typed in step 7.

RESULTS

1	Ns(t) — survivor regression	0.0001498·t² -0.078415·t +9.979959
2	Nf(t) = N0 − Ns(t)	-0.0001498·t² +0.078415·t +0.020041
3	Regression R²	0.99769
4	λ(t) = Nf'(t)/Ns(t)	(-0.0002996·t +0.078415) / (0.0001498·t² -0.078415·t +9.979959)
5	MTBF = ∫ R(t) dt	83.12
6	λ(0) / λ(mid-grid, t = 110)	0.0078572857 / 0.0143575158
7	R(70) = e^{-H}, H = ∫λ dt = 0.64717	0.5235 (52.35%)

Results

- ① Ns(t) — survivor regression 0.0001498·t² -0.078415·t +9.979959: the fitted number of pumps still running at age t.
- ② Nf(t) = N0 − Ns(t) -0.0001498·t² +0.078415·t +0.020041: the fitted number failed by age t - the same curve mirrored.
- ③ Regression R² 0.99769: the curve follows the survivor table closely.
- ④ λ(t) = Nf'(t)/Ns(t) (-0.0002996·t +0.078415) / (0.0001498·t² -0.078415·t +9.979959): the failure rate at age t.
- ⑤ MTBF = ∫ R(t) dt 83.12: the mean life of a pump, in operating hours.
- ⑥ λ(0) / λ(mid-grid, t = 110) 0.0078572857 / 0.0143575158: the rate when new and at mid-life - it nearly doubles.
- ⑦ R(70) = e−H, H = ∫λ dt = 0.64717 0.5235 (52.35%): about half the pumps survive to 70 hours.

INTERPRETATION

1	The exponential form R(t) = $e^{-\int\lambda(u)du}$ is the general time-dependence model: it covers constant AND non-constant failure rates. The failure rate is estimated from the fleet itself — λ(t) = Nf'(t)/Ns(t) from the regression of the survivor table — so no rate shape is assumed in advance. Here λ(t) INCREASES with age — wear-out (Zone C of the bathtub curve); preventive maintenance can pay. MTBF = ∫R dt = 83.12 (integration stops at t = 218, where the fitted survivor curve dies). The χ^2 exponential test below could NOT reject a constant rate — with this sample size both readings are defensible; compare them.

Interpretation

- ① ReliaEn's reading: The exponential form R(t) = e−∫λ(u)du is the general time-dependence model: it covers constant AND non-constant failure rates. The failure rate is estimated from the fleet itself — λ(t) = Nf'(t)/Ns(t) from the regression of the survivor table — so no rate shape is assumed in advance. Here λ(t) INCREASES with age — wear-out (Zone C of the bathtub curve); preventive maintenance can pay. MTBF = ∫R dt = 83.12 (integration stops at t = 218, where the fitted survivor curve dies). The χ^2 exponential test below could NOT reject a constant rate — with this sample size both readings are defensible; compare them. The curve says wear-out; the pre-test says the data cannot prove it.

EXPONENTIAL TEST — IS THE FAILURE RATE CONSTANT? (χ^2)

1. χ^2_0 = 2.7138 ≤ 11.0705 — the test CANNOT reject a constant failure rate. This is not proof of constancy: with only 10 failures the expected count in each interval is small, so the test has little power to detect a gradually rising rate — read it together with the λ(t) trend below.

	Quantity	Value
2	λ estimate (60% interval upper bound)	0.0109235
3	χ^2_0 — test statistic	2.7138
4	χ^2 critical (α = 5%, dof = 5)	11.0705

Exponential test — is the failure rate constant? (χ^2)

- ① χ^2_0 = 2.7138 ≤ 11.0705 — the test CANNOT reject a constant failure rate. This is not proof of constancy: with only 10 failures the expected count in each interval is small, so the test has little power to detect a gradually rising rate — read it together with the λ(t) trend below.
- ② λ estimate (60% interval upper bound) 0.0109235: the constant rate the test compares against.
- ③ χ^2_0 — test statistic 2.7138.
- ④ χ^2 critical (α = 5%, dof = 5) 11.0705: χ^2_0 would have to exceed this to reject a constant rate.

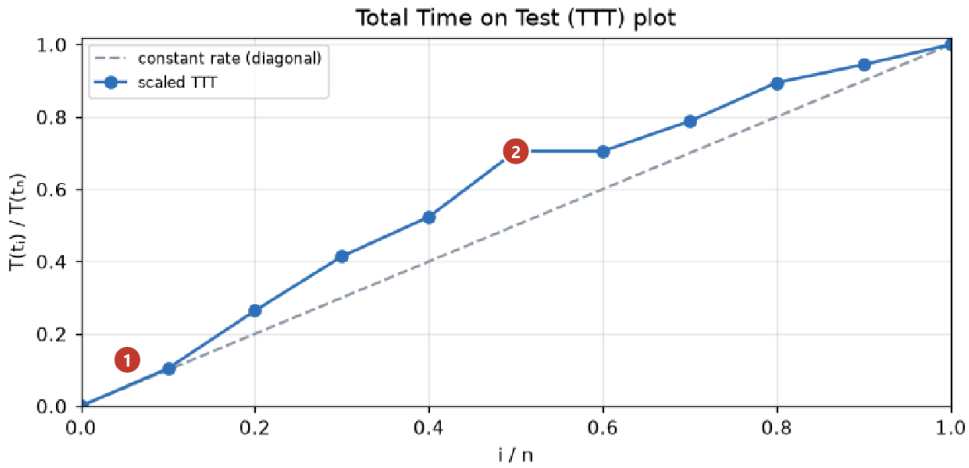
BARLOW–PROSCHAN TEST — TREND IN THE FAILURE RATE (TTT)

1. |z| = 0.9735 ≤ 1.96 — the test cannot reject a constant rate; the statistic points toward an increasing rate (p = 0.330). On the TTT plot below, bowing ABOVE the diagonal means an increasing rate.

	Quantity	Value
2	S — Σ scaled TTT (i = 1..n−1)	5.3431
3	E[S] under constant rate	4.5
4	z — normal approximation	0.9735
5	p-value (two-sided)	0.3303

Barlow–Proschan test — trend in the failure rate (TTT)

- ① |z| = 0.9735 ≤ 1.96 — the test cannot reject a constant rate; the statistic points toward an increasing rate (p = 0.330). On the TTT plot below, bowing ABOVE the diagonal means an increasing rate.
- ② S — Σ scaled TTT (i = 1..n−1) 5.3431.
- ③ E[S] under constant rate 4.5: S above it points toward an increasing rate.
- ④ z — normal approximation 0.9735.
- ⑤ p-value (two-sided) 0.3303.



Interactive Plot

The total time on test plot

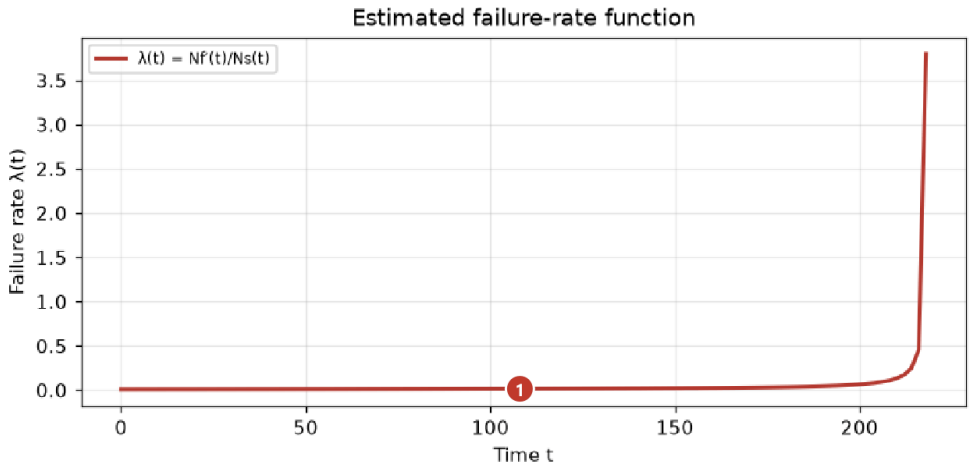
- ① constant rate (diagonal): where the points would lie under a constant rate.
- ② scaled TTT: the pumps' own curve, bowing slightly above the diagonal - the direction of wear-out, too little to prove it.



Interactive Plot

The survivor and failure curves

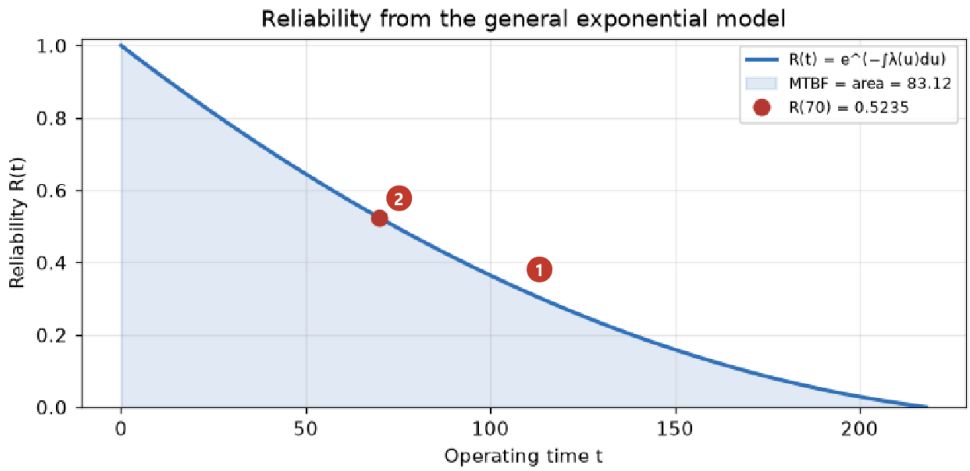
- ① Ns(t) survivors: the survivor table, point by point.
- ② Ns fit (R^2 = 0.9977): the fitted survivor curve.
- ③ Nf(t) failures: the failures so far, point by point.
- ④ Nf fit (mirror): the fitted failure curve, the mirror of ②.



Interactive Plot

The failure rate against age

- ① λ(t) = Nf'(t)/Ns(t): the failure rate rising with age.



Interactive Plot

Reliability against age

- ① R(t) = e^(−∫λ(u)du): the share of pumps still running at each age.
- ② R(70) = 0.5235: the point at the target age of step 7.

SURVIVOR TABLE — DERIVED FROM THE EVENT TIMES

1 All units start together at t = 0; every failure or censoring removes one unit. Check this table against your records — each simultaneous failure needs its own entry in the worksheet.

2 t	3 Ns(t)	4 Nf(t)
0	10	0
10	9	1
27	8	2
45	7	3
60	6	4
89	4	6
109	3	7
143	2	8
167	1	9
220	0	10

Survivor table — derived from the event times

- ① All units start together at t = 0; every failure or censoring removes one unit. Check this table against your records — each simultaneous failure needs its own entry in the worksheet.
- ② t: each age at which units left the fleet.
- ③ Ns(t): the units still running; at 89 it drops from 6 to 4, because two pumps failed together.
- ④ Nf(t): the units failed so far.

REGRESSION USABILITY (COURSE RULE)

1 $R^2 = 0.99769 \geq 0.6$ — the fitted model is usable.

2 The multiple correlation of the Ns(t)/Nf(t) regression must be at least 0.6 before the estimated λ(t) may be used (Day 2 course rule).

Regression usability (course rule)

- ① $R^2 = 0.99769 \geq 0.6$ — the fitted model is usable.
- ② The multiple correlation of the Ns(t)/Nf(t) regression must be at least 0.6 before the estimated λ(t) may be used (Day 2 course rule).

1 Licensed to ReliaEn Training · RL-2026-0017

The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

Part 5 — How to interpret the results

Check R^2 first. R^2 = 0.99769 is far above the course rule of 0.6, so λ(t) may be used. Below 0.6 the report adds a caution card, and every number is for reference only.

Two readings of the same pumps. The fitted curve says the failure rate nearly doubles from new to mid-life, from 0.0078572857 to 0.0143575158 per hour: wear-out. The two pre-tests, which read the raw ages, cannot reject a constant rate: χ^2_0 = 2.7138 stays below 11.0705, and the Barlow–Proschan z = 0.9735 stays below 1.96, although it points toward an increasing rate (p = 0.330). With ten failures both readings are defensible, and the report says so. Before you spend money on preventive replacement, gather more failures or fit the Weibull model and read its shape interval.

The mean life and the target. The mean life is 83.12 operating hours: the area under R(t) up to 218 hours, where the fitted survivor curve reaches zero. About half the pumps (R = 0.5235) survive to 70 hours.

What follows for maintenance. No cost card is printed: the example enters no costs, and the report computes a replacement age only when the costs are entered and the pre-tests show a rising rate. Here they do not, so a replacement age from the curve alone would rest on where the quadratic happens to stop, not on proven wear-out.