Gamma process — life from measured wear, before any unit fails

Menu: Life Data ▸ Degradation ▸ Gamma process...

Part 1 — What the function does

What it calculates. Some units do not fail suddenly: they wear, and you can measure the wear - a liner's thickness lost, a crack's length, a bearing's vibration. The gamma process models the cumulative wear X(t) as random growth that never goes back: in each interval it grows by a gamma-distributed step, X(t) ~ Gamma(v·t, u), with mean (v/u)·t. A unit fails when its wear first passes a limit D. ReliaEn fits the shape rate v and the step-size rate u from the wear increments of a fleet, by maximum likelihood with bounds, and turns them into a life distribution: the B10 life, the median, the MTTF and a reliability ladder - without any unit having to fail. It also tests whether the wear speeds up, and projects each unit at its own pace.

What ReliaEn prints	What it tells you
v̂ and **û** with bounds	How the wear grows: its mean rate v/u and its step noise 1/u
Mean life ≈ uD/v, **B10**, **Median**, **MTTF**	When a unit's wear reaches the limit D
Reliability ladder	R(t) and its one-sided lower bound for planning, at ages around the mean life
Degradation trend	Whether the wear speeds up with age (t^β against linear)
Unit projections	When each unit, at its own pace, reaches D

From which data. A time column and one column per unit with its cumulative wear at each inspection; a blank cell is a missed inspection. The limit D is typed in the form, in the same unit as the readings.

When an engineer uses it. When the wear of a fleet is measured at inspections and the question is when units will reach their wear limit - before any has failed - and how reliability falls with age.

When not to use it.

Situation	Use instead
The wear can go down as well as up (a measurement that drifts both ways)	Life Data ▸ Degradation ▸ Wiener process...
Only failure ages are known, no wear readings	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution...

Part 2 — The example's input data

Setting. A test rig measures the cumulative wear of six pump liners, B to G, at eight inspections from age 10 to age 100. The wear limit is 10. The reliability engineer wants the liners' life at that limit and how reliability falls with age.

Input	Value
Inspection ages (8)	10, 20, 30, 40, 55, 70, 85, 100
Wear readings	six liners, eight readings each, 3.08 to 41.72
File in the worked-example zip	2-Gamma-Process/Example-1.csv — seven columns, headings Time, Unit B ... Unit G, 8 rows

Failure threshold D	10 — typed in the form
Confidence %	95 — the value the form opens with

	① A	② B	③ C	④ D	⑤ E	⑥ F	⑦ G	H
1	10	5.48	3.08	4.27	3.63	4.68	3.32	
2	20	6.76	9.0	9.49	7.6	7.04	5.43	
3	30	9.09	12.8	15.03	12.64	8.81	11.14	
4	40	14.51	15.51	18.56	16.15	12.93	13.5	
5	55	21.53	21.48	25.47	19.78	17.11	18.86	
6	70	27.87	24.99	29.95	24.97	22.27	23.25	
7	85	34.02	31.45	35.19	30.04	26.69	30.3	
8	100	38.52	39.6	41.72	37.68	30.93	34.44	
9								

The example data in the worksheet

- ① Column A: the inspection ages, 10 to 100.
- ② to ⑦ Columns B to G: the cumulative wear of liners B to G at each age.

Part 3 — How to use it, step by step

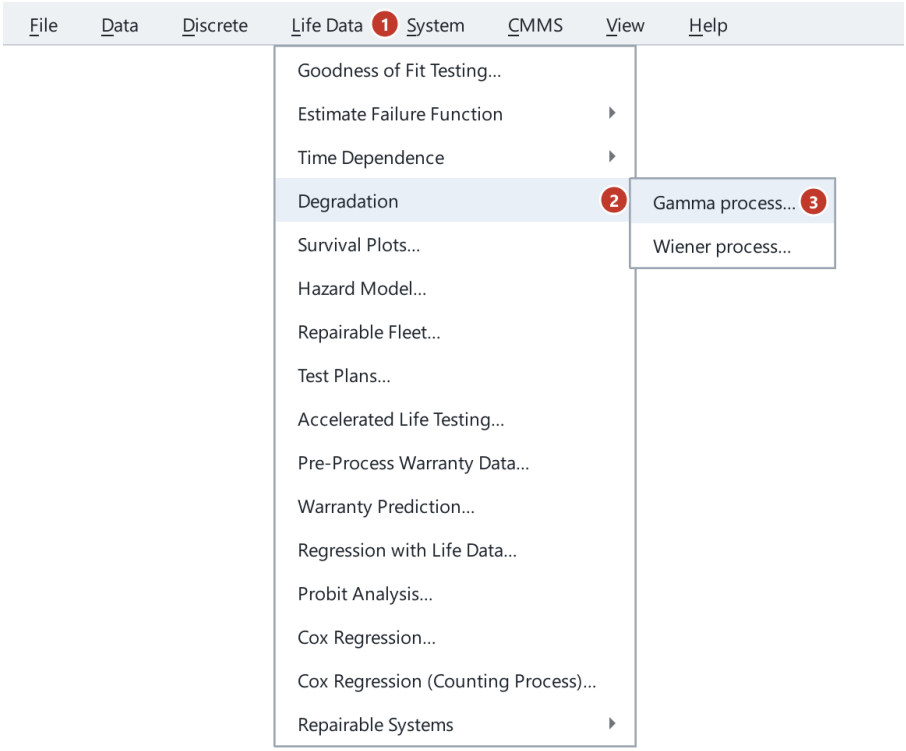
1. **Put the time and wear columns in A to G.** Copy the seven columns of Example-1.csv, Time and Unit B to Unit G, into columns A to G of the worksheet, starting at row 1.

	① A	② B	③ C	④ D	⑤ E	⑥ F	⑦ G	H
1	10	5.48	3.08	4.27	3.63	4.68	3.32	
2	20	6.76	9.0	9.49	7.6	7.04	5.43	
3	30	9.09	12.8	15.03	12.64	8.81	11.14	
4	40	14.51	15.51	18.56	16.15	12.93	13.5	
5	55	21.53	21.48	25.47	19.78	17.11	18.86	
6	70	27.87	24.99	29.95	24.97	22.27	23.25	
7	85	34.02	31.45	35.19	30.04	26.69	30.3	
8	100	38.52	39.6	41.72	37.68	30.93	34.44	
9								

Step 1 — the ages and the wear readings

- ① Column A: the eight inspection ages.
- ② to ⑦ Columns B to G: the six liners' wear readings.

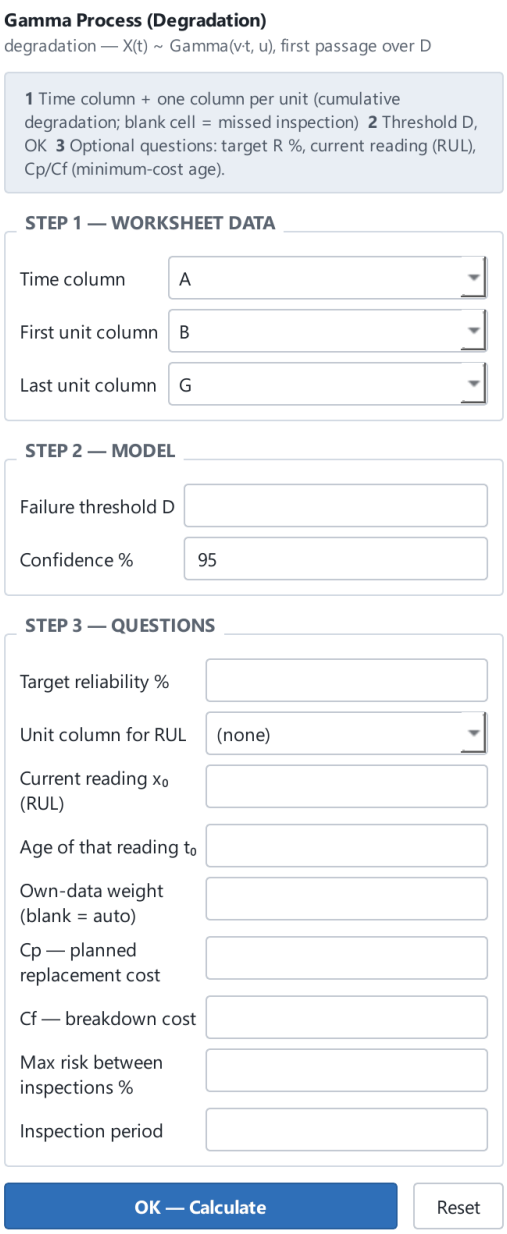
2. **Choose Life Data ▸ Degradation ▸ Gamma process…** in the menu bar. The form Gamma Process (Degradation) opens in the left panel.



Step 2 — the menu path

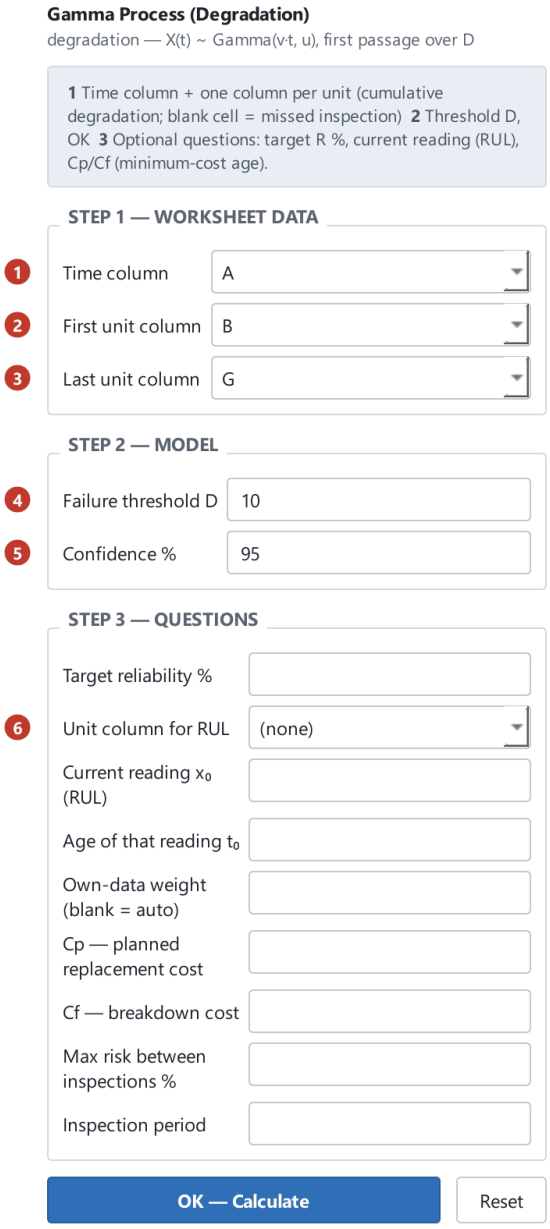
- ① Life Data: the menu.
- ② Degradation: the submenu.
- ③ Gamma process…: the entry to click.

The form as it opens, before any box is set:



Step 2 — the form as it opens

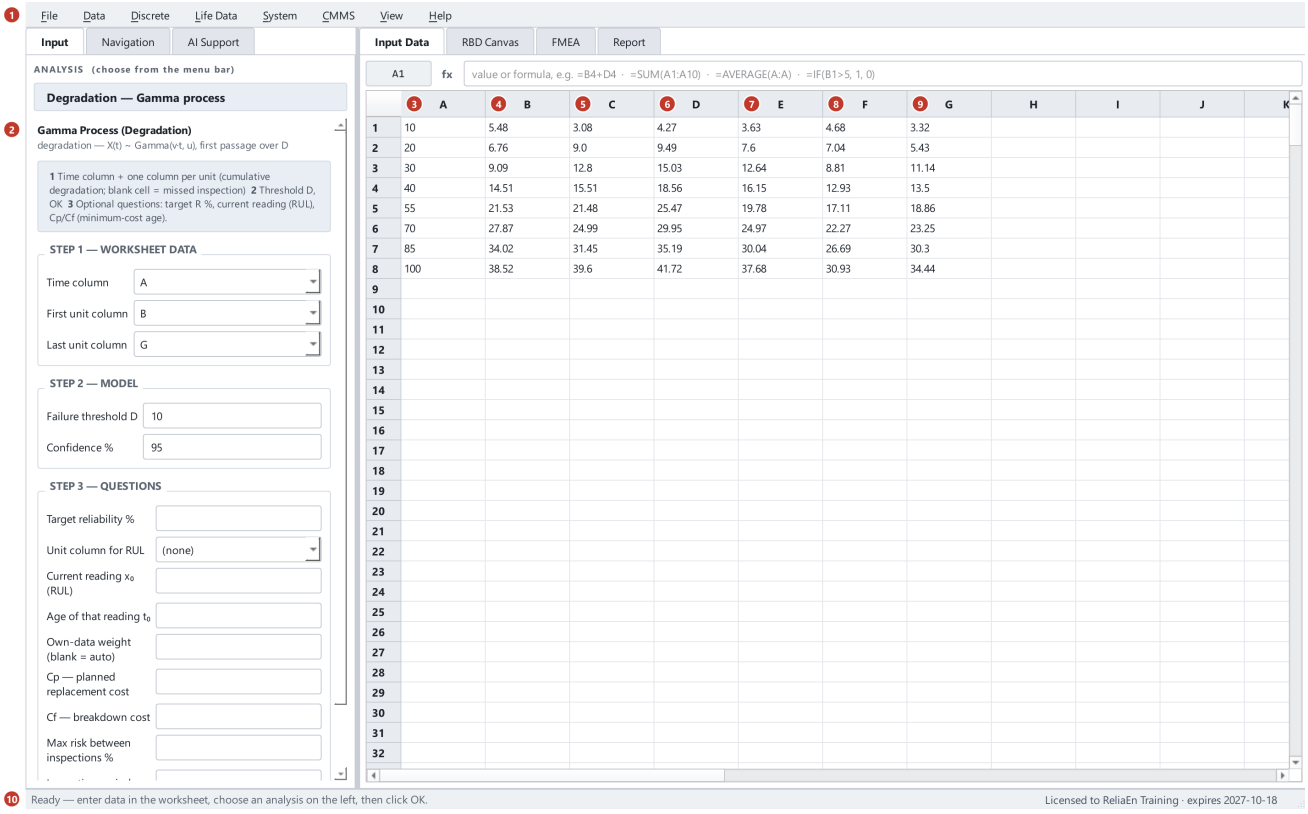
Steps 3 to 8 set or check the form's six boxes, marked ① to ⑥ in the picture:



Steps 3 to 8 — the form set for this example

3. ① **In Time column, choose A.**
4. ② **In First unit column, choose B.**
5. ③ **In Last unit column, choose G.** Columns B to G are the six liners.
6. ④ **In Failure threshold D, type 10.** The wear limit, in the readings' unit.
7. ⑤ **Check that Confidence % reads 95.**
8. ⑥ **Leave Unit column for RUL at (none).** The other optional questions under STEP 3 — QUESTIONS stay empty too.

9. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates and shows the report.



Step 9 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 8; OK — Calculate at its foot.
- ③ Column A: the inspection ages.
- ④ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — Degradation — Gamma Process

```
2026-10-30 19:14:11
X(t) ~ Gamma(v·t, u) ; F_T(t) = Q(vt, uD) ; R(t) = 1 − F_T(t)
```

The report's head

- The title Life Data — Degradation — Gamma Process, the date and time of the run, and the model in one line: X(t) ~ Gamma(v·t, u); F_T(t) = Q(vt, uD); R(t) = 1 − F_T(t).

INPUT DATA		
1	Units / inspections	6 / 8 times
2	n — increments used	48
3	D — failure threshold	10
4	Inspection intervals	unequal Δt (blanks widen the gap)

Input data

- ① Units / inspections 6 / 8 times.
- ② n — increments used 48: six liners × eight inspections, each read against the one before (the first against zero wear at age 0).
- ③ D — failure threshold 10.
- ④ Inspection intervals unequal Δt (blanks widen the gap): the gaps are 10, then 15 from age 40.

RESULTS		
1	v̂ — shape rate	0.889059 [0.599544, 1.31838]
2	û — step-size rate	2.39327 [1.5994, 3.58116]
3	Mean degradation rate v/u	0.371483
4	Step-noise measure Var/Mean = 1/u	0.417839
5	Mean life ≈ uD/v	26.92
6	B10 life (exact first passage)	20.5651
7	Median life (exact)	27.2931
8	MTTF (exact ∫R)	27.4815

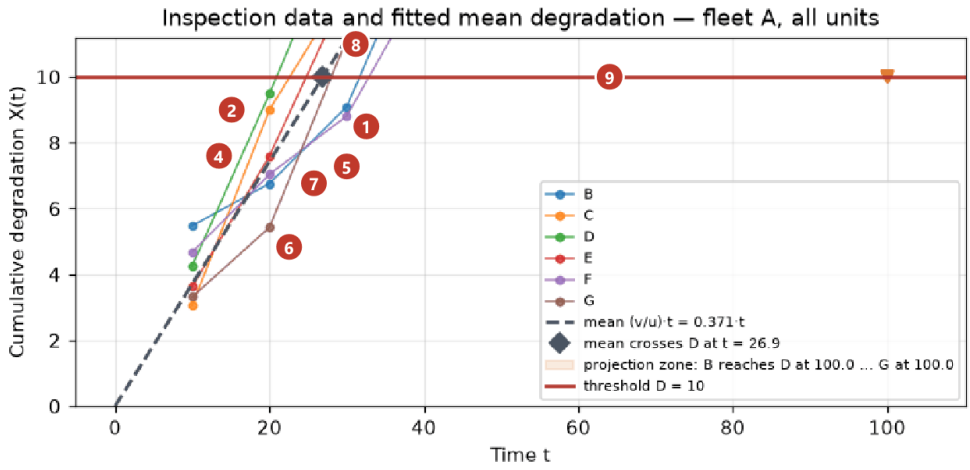
Results

- ① v̂ — shape rate 0.889059 [0.599544, 1.31838].
- ② û — step-size rate 2.39327 [1.5994, 3.58116].
- ③ Mean degradation rate v/u 0.371483: the liners wear about 0.37 per unit of time on average.
- ④ Step-noise measure Var/Mean = 1/u 0.417839: how irregular the steps are.
- ⑤ Mean life ≈ uD/v 26.92: the age at which the expected wear reaches D.
- ⑥ B10 life (exact first passage) 20.5651: one liner in ten reaches the limit by this age.
- ⑦ Median life (exact) 27.2931: half the liners have reached it.
- ⑧ MTTF (exact ∫R) 27.4815: the mean age at which a liner reaches the limit.

INTERPRETATION	
1	Cumulative degradation is modeled as X(t) ~ Gamma(v·t, u): monotone random growth with mean (v/u)·t. Failure is the first passage over D, so the life distribution is closed form — R(t) = 1 − Q(vt, uD) — and the hazard rate rises naturally with age (wear-out emerges from accumulation, not from an assumption). No unit needs to fail for this fit: the 48 inspection increments carry the information.

Interpretation

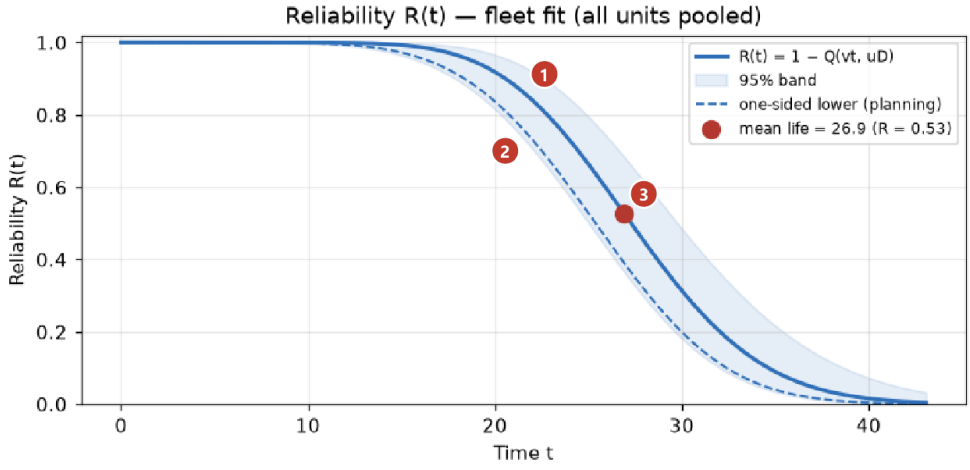
- ① ReliaEn's reading: Cumulative degradation is modeled as X(t) ~ Gamma(v·t, u): monotone random growth with mean (v/u)·t. Failure is the first passage over D, so the life distribution is closed form — R(t) = 1 − Q(vt, uD) — and the hazard rate rises naturally with age (wear-out emerges from accumulation, not from an assumption). No unit needs to fail for this fit: the 48 inspection increments carry the information.



Interactive Plot

The wear paths

- ① to ⑥ B to G: each liner's measured wear path.
- ⑦ mean (v/u)·t = 0.371·t: the fitted mean wear line.
- ⑧ mean crosses D at t = 26.9: where the mean line reaches the limit - the mean life.
- ⑨ threshold D = 10: the wear limit.



Interactive Plot

Reliability against age

- ① R(t) = 1 − Q(vt, uD): the share of liners still below the limit at each age.
- ② one-sided lower (planning): the lower bound to plan with.
- ③ mean life = 26.9 (R = 0.53): at the mean life, a little more than half are still below D.

ESTIMATION — METHOD OF MOMENTS START, MLE REFINED (FLEET FIT)

1 û = v·ΣΔt/ΣΔx is exact (profile); v solves ln v − ψ(v) = ln x̄ − mean(ln Δx) — a single root because the left side is strictly decreasing.

2 Step	3 v	4 u	5 ln-likelihood
6 Method of Moments	10.76	28.87	-292.635
7 Maximum likelihood (reported)	0.8891	2.393	-82.041

Estimation — Method of Moments start, MLE refined (fleet fit)

- ① û = v·ΣΔt/ΣΔx is exact (profile); v solves ln v − ψ(v) = ln x̄ − mean(ln Δx) — a single root because the left side is strictly decreasing. The equation printed is the one for equal gaps between inspections. With the unequal gaps of this example (10, then 15 from age 40), ReliaEn solves the general equation Σ Δt·[ln u + ln Δx − ψ(v·Δt)] = 0, of which the printed one is the special case.
- ② to ⑤ The columns Step, v, u, ln-likelihood.
- ⑥ Method of Moments: v 10.76, u 28.87, ln-likelihood -292.635 - a starting value only, which the maximum-likelihood fit replaces; no result on the page uses it.
- ⑦ Maximum likelihood (reported): v 0.8891, u 2.393, ln-likelihood -82.041 - the fit the results use.
- A later release corrects the starting value for unequal gaps and the note above.

DEGRADATION TREND — LINEAR VS T^B (LIKELIHOOD-RATIO TEST)

1 p = 0.7092 ≥ 0.05 — acceleration NOT detected: the simpler linear model is reported (β̂ = 0.983, its interval includes 1). This is not proof of linearity — a mild curve may need more inspection data to show.

Quantity	Value
2 β̂ — free trend exponent	0.9829 [0.894, 1.072]
3 LR = 2(lnL_β − lnL_linear)	0.1391
4 p-value (χ², 1 dof)	0.7092
5 Model reported	linear (β = 1)

6 Nested models: β = 1 inside free β; LR ~ $\chi^2(1)$. With β confirmed, parameter bounds treat β as fixed at β̂.

Degradation trend — linear vs t^β (likelihood-ratio test)

- ① p = 0.7092 ≥ 0.05 — acceleration NOT detected: the simpler linear model is reported (β̂ = 0.983, its interval includes 1). This is not proof of linearity — a mild curve may need more inspection data to show.
- ② β̂ — free trend exponent 0.9829 [0.894, 1.072].
- ③ LR = 2(lnL_β − lnL_linear) 0.1391.
- ④ p-value (χ^2, 1 dof) 0.7092.
- ⑤ Model reported linear (β = 1).
- ⑥ Nested models: β = 1 inside free β; LR ~ $\chi^2(1)$. With β confirmed, parameter bounds treat β as fixed at β̂.

RELIABILITY LADDER — FLEET FIT (A RANDOMLY CHOSEN UNIT)

1 R(t) = P(X(t) < D) — probability a random unit has not yet crossed the threshold at age t. The lower column is the one-sided 95% bound used for planning (CONVENTIONS row 11).

2 Time t	3 R(t)	4 R lower (one-sided)	5 F(t)
10.77	0.9997	0.9972	0.0003
13.46	0.9974	0.9867	0.0026
16.15	0.9861	0.9551	0.0139
18.84	0.9486	0.8839	0.0514
20.19	0.9126	0.8284	0.0874
21.54	0.8619	0.7588	0.1381
24.23	0.7153	0.5841	0.2847
26.92	0.5272	0.3900	0.4728
32.30	0.1881	0.1032	0.8119

Reliability ladder — fleet fit (a randomly chosen unit)

- ① R(t) = P(X(t) < D) — probability a random unit has not yet crossed the threshold at age t. The lower column is the one-sided 95% bound used for planning (CONVENTIONS row 11).
- ② to ⑤ The columns Time t, R(t), R lower (one-sided), F(t). The ages are steps of the mean life: at 20.19, R is 0.9126 (lower 0.8284); at 26.92, 0.5272 (lower 0.3900); at 32.30, 0.1881 (lower 0.1032).

UNIT PROJECTIONS — EACH UNIT AT ITS OWN PACE

1 Mean-trend projection: each unit continues from its LAST reading at its OWN wear-clock rate (fastest first). This is the spread shown as the shaded zone on the paths plot.

2 Unit	3 Last reading	4 Projected age at D	5 Remaining life
6 B	38.52 @ t = 100	already past D	—
7 C	39.6 @ t = 100	already past D	—
8 D	41.72 @ t = 100	already past D	—
9 E	37.68 @ t = 100	already past D	—
10 F	30.93 @ t = 100	already past D	—
11 G	34.44 @ t = 100	already past D	—

12 A deterministic projection of each unit's average pace — for the probability distribution of one unit's remaining life, use the RUL question (Step 3) with that unit's current reading.

Unit projections — each unit at its own pace

- ① Mean-trend projection: each unit continues from its LAST reading at its OWN wear-clock rate (fastest first). This is the spread shown as the shaded zone on the paths plot.
- ② to ⑤ The columns Unit, Last reading, Projected age at D, Remaining life.
- ⑥ to ⑪ Liners B to G: every one is already past D at its last reading at age 100 (30.93 to 41.72) - the rig kept running past the limit.
- ⑫ A deterministic projection of each unit's average pace — for the probability distribution of one unit's remaining life, use the RUL question (Step 3) with that unit's current reading.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

Part 5 — How to interpret the results

What the wear says. The liners wear about 0.371483 per unit of time on average (v/u), in irregular steps (1/u = 0.417839), and the trend test finds no speed-up (p = 0.7092): the wear is linear in age. No liner had to fail for this - the 48 measured increments carry the information.

When the liners reach the limit. One liner in ten reaches the wear limit 10 by age 20.5651 (B10); half by 27.2931 (the median); the mean is 27.4815. The mean life uD/v = 26.92 is simply the age at which the expected wear reaches D.

How fast reliability falls late in life. Between ages 20.19 and 32.30, R falls from 0.9126 to 0.1881. Plan with the one-sided lower column: at age 20.19 it is 0.8284, not 0.9126. Every extra period late in life costs much more reliability than one early on.