ReliaEn 1.0.0 User Manual · Chapter 2.12

Lognormal — life as multiplicative damage, read on the log of the data

Menu: Life Data ▸ Time Dependence ▸ Lognormal distribution...

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# Part 1 — What the function does

**What it calculates.** The lognormal model says the logarithm of a unit's life is normally distributed: ln T ~ Normal(μ, σ). It fits damage that grows in proportion to its own size - a fatigue crack, a repair job that drags on. ReliaEn fits μ (the mean of ln t) and σ (the spread of ln t) by maximum likelihood, with bounds, and gives the median e^μ, the mean and the mode, the B-lives with bounds and the service intervals. It checks the fit four ways (probability plot, PPCC, two formal tests on ln t), compares it with a Weibull by AIC, and prices two improvements: more tests, and less scatter.

| What ReliaEn prints | What it tells you |
|---|---|
| **μ̂** and **σ̂** with bounds | The centre and the spread of ln t - not of t itself |
| **Median**, **Mean (MTTF)**, **Mode** | Three centres, always ordered mode < median < mean |
| **B-lives** with bounds, **R at a chosen time** | By what age a share of units has failed; the share that survives a chosen age |
| **PPCC**, **Shapiro–Wilk**, **Anderson–Darling** | Whether the lognormal model is rejected |
| **Competitor model** | Whether a Weibull explains the data as well, by AIC |
| **Value of more data** and **the σ lever** | What more tests, or a narrower scatter, would do to B10 |

**From which data.** The failure ages in one worksheet column, one row per failed unit, each above 0 (the logarithm refuses 0); survivor ages, if any, in a different column. The same form also reads repair times for maintainability work.

**When an engineer uses it.** When failure comes from damage that multiplies - fatigue cracks, corrosion pits that grow - and the question is how long units last and what would lengthen their life.

**When not to use it.**

| Situation | Use instead |
|---|---|
| Failure comes from the weakest of many flaws | Life Data ▸ Time Dependence ▸ Weibull (2P) distribution... |
| Failure comes from accumulated damage events | Life Data ▸ Time Dependence ▸ Gamma distribution... |
| Pure wear-out with a hazard that keeps rising | Life Data ▸ Time Dependence ▸ Weibull (2P) distribution..., or Life Data ▸ Degradation ▸ Gamma process... |

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# Part 2 — The example's input data

*Setting.* A gearbox maker runs 12 shafts to fatigue failure on a test rig, in thousands of cycles. The reliability engineer wants their life, how many shafts survive 100 thousand cycles, and what would lengthen the B10 life.

| Input | Value |
|---|---|
| Failure ages, thousands of cycles (12) | 22.0, 31.5, 38.4, ... (the 12 values of the file) |
| File in the worked-example zip | 2-Lognormal/Example-1.csv — one column, heading Failure times, 12 rows |

| | |
|---|---|
| Evaluate R/M at t (optional) | 100 — typed in the form |
| Confidence % / Level % | 95 / 10 — the values the form opens with |

| | ① A | B |
|---|---|---|
| 1 | 22.0 | |
| 2 | 31.5 | |
| 3 | 38.4 | |
| 4 | 45.0 | |
| 5 | 52.7 | |
| 6 | 61.2 | |
| 7 | 70.9 | |
| 8 | 82.3 | |
| 9 | 96.0 | |
| 10 | 114.5 | |
| 11 | 141.8 | |
| 12 | 190.2 | |
| 13 | | |

The example data in the worksheet

- ① Column A: the 12 failure ages, one per row, in thousands of cycles.

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# Part 3 — How to use it, step by step

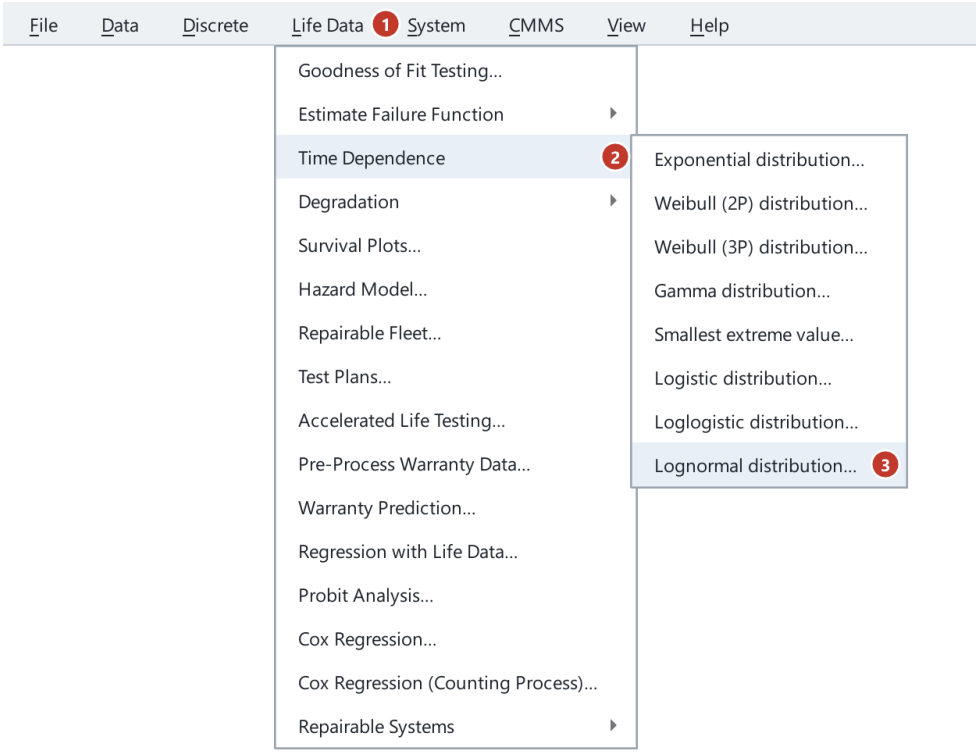
1. **Put the failure ages in column A.** Copy the 12 values of Example-1.csv that stand under its heading Failure times into column A of the worksheet, starting at row 1.

| | ① A | B |
|---|---|---|
| 1 | 22.0 | |
| 2 | 31.5 | |
| 3 | 38.4 | |
| 4 | 45.0 | |
| 5 | 52.7 | |
| 6 | 61.2 | |
| 7 | 70.9 | |
| 8 | 82.3 | |
| 9 | 96.0 | |
| 10 | 114.5 | |
| 11 | 141.8 | |
| 12 | 190.2 | |
| 13 | | |

Step 1 — the failure ages in column A

- ① Column A: the 12 failure ages, 22.0 to 190.2.

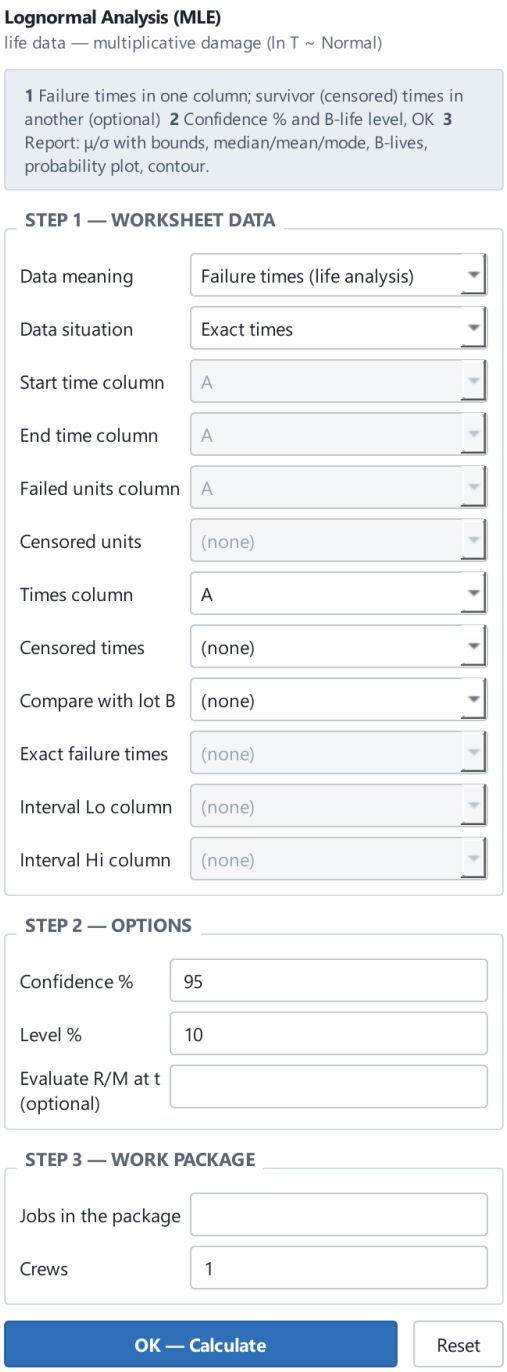
2. **Choose Life Data ▸ Time Dependence ▸ Lognormal distribution…** in the menu bar. The form Lognormal Analysis (MLE) opens in the left panel.




Step 2 — the menu path

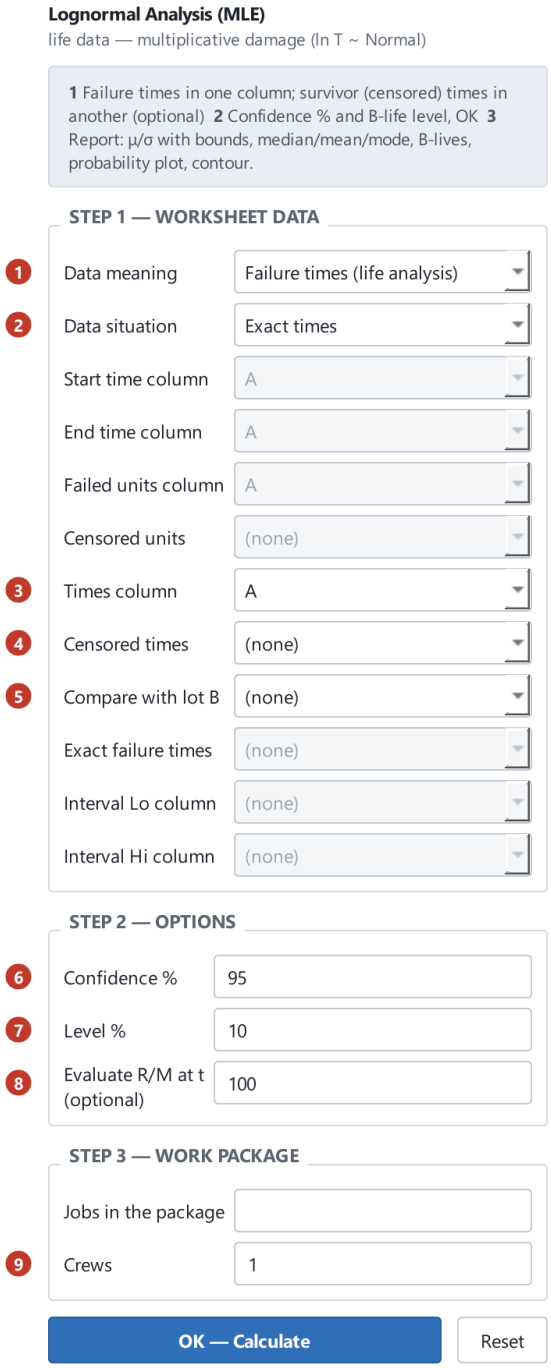
- ① Life Data: the menu.
- ② Time Dependence: the submenu.
- ③ Lognormal distribution…: the entry to click.

The form as it opens, before any box is set:




Step 2 — the form as it opens

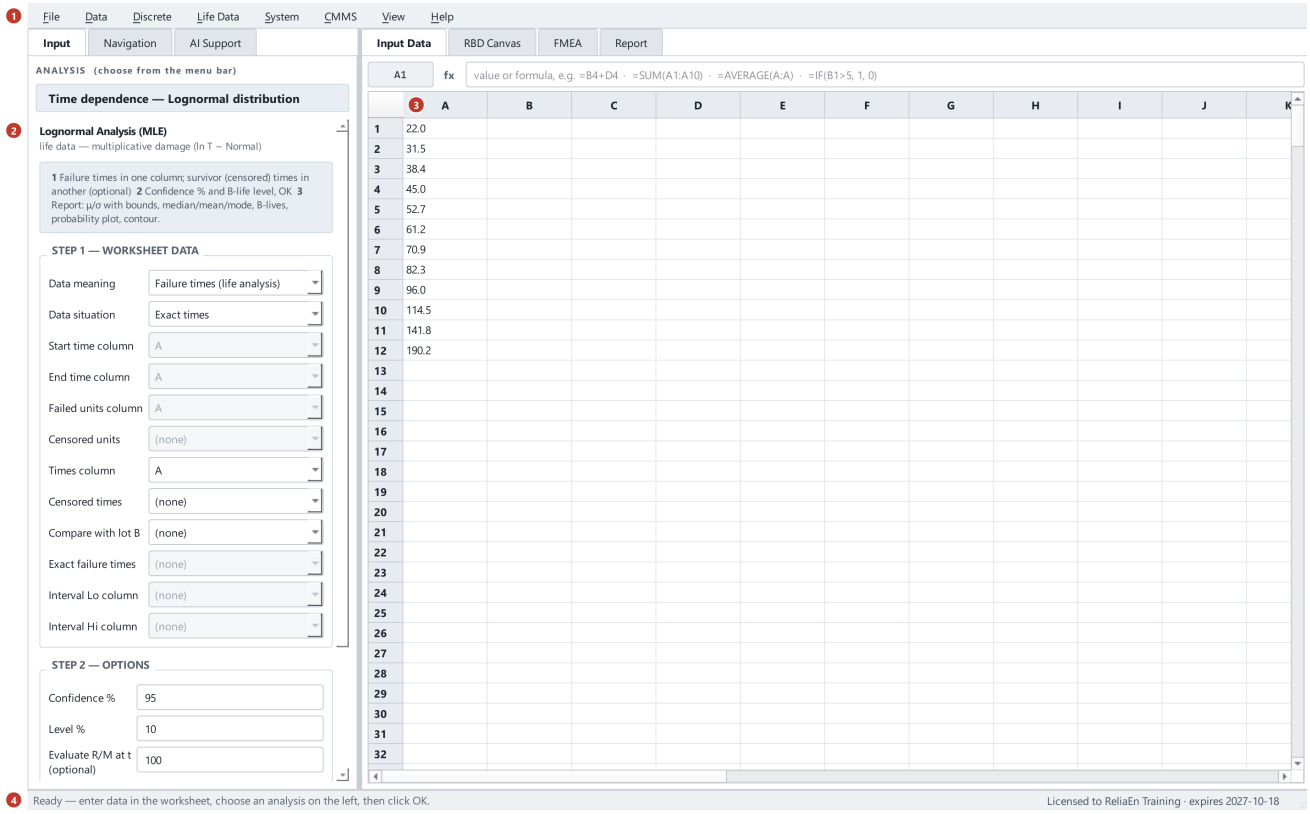
Steps 3 to 11 set or check the form's nine boxes, marked ① to ⑨ in the picture:




Steps 3 to 11 — the form set for this example

3. ① **Check that Data meaning reads Failure times (life analysis).**
4. ② **Check that Data situation reads Exact times.**
5. ③ **In Times column, choose A.**
6. ④ **Leave Censored times at (none).** Every shaft failed.
7. ⑤ **Leave Compare with lot B at (none).** No second lot.
8. ⑥ **Check that Confidence % reads 95.**
9. ⑦ **Check that Level % reads 10.** The B-life in the results.
10. ⑧ **In Evaluate R/M at t (optional), type 100.**
11. ⑨ **Leave Crews at 1.** Repair mode only.

12. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn calculates and shows the report.



Step 12 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 11; OK — Calculate at its foot.
- ③ Column A: the 12 failure ages.
- ④ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

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# Part 4 — The report, card by card

**Life Data — Lognormal (2P) — Time Dependence**

2026-10-30 19:08:40

ln T ~ Normal(μ, σ²) ; R(t) = 1 − Φ((ln t − μ)/σ) ; $B_\alpha$ = e^(μ + z_α σ)

The report's head

- The title Life Data — Lognormal (2P) — Time Dependence, the date and time of the run, and the model in one line: ln T ~ Normal(μ, $\sigma^2$); R(t) = 1 − Φ((ln t − μ)/σ); B_α = e^(μ + z_α σ).

| | INPUT DATA | |
|---|---|---|
| 1 | n — units on test | 12 |
| 2 | r — failures | 12 |
| 3 | censored (survivors) | 0 |
| 4 | Estimation | closed form (complete data) |

Input data

- ① n — units on test 12.
- ② r — failures 12. Check r first: a blank cell is skipped.
- ③ censored (survivors) 0.
- ④ Estimation closed form (complete data): with every unit failed, μ̂ is the mean of ln t and σ̂ its spread - no search is needed.

| | RESULTS | |
|---|---|---|
| 1 | μ̂ — log-mean | 4.1878 [3.84313, 4.53247] |
| 2 | σ̂ — log-SD (MLE, ÷n) | 0.609183 [0.408316, 0.908863] (unbiased s = 0.63627) |
| 3 | Median = e^μ | 65.8775 |
| 4 | Mean (MTTF) = e^(μ+σ²/2) | 79.3088 |
| 5 | Mode = e^(μ−σ²) | 45.4537 |
| 6 | B10 life | 30.1776 [18.953, 48.0498] |
| 7 | R(100) — survival | 0.2466292029 |

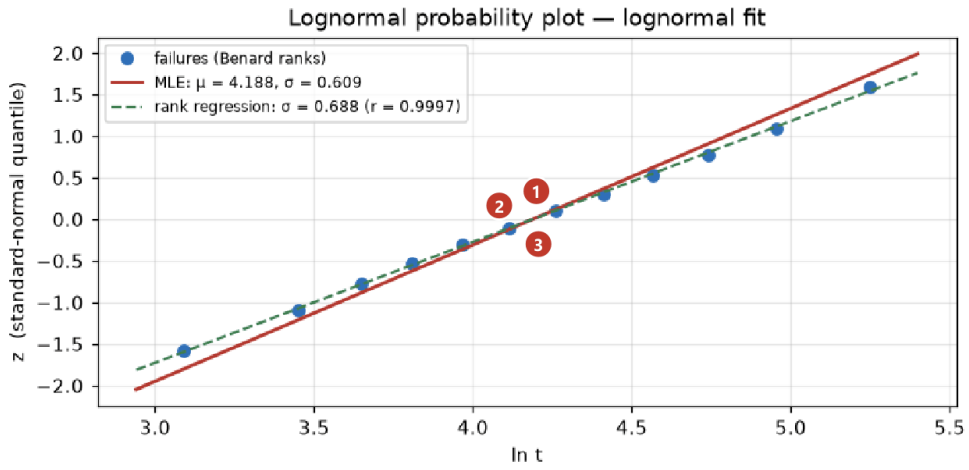
Results

- ① μ̂ — log-mean 4.1878 [3.84313, 4.53247]: the centre of ln t, in log thousands of cycles.
- ② σ̂ — log-SD (MLE, ÷n) 0.609183 [0.408316, 0.908863] (unbiased s = 0.63627): the spread of ln t.
- ③ Median = e^μ 65.8775: half the shafts fail by 65.9 thousand cycles.
- ④ Mean (MTTF) = e^(μ+σ²/2) 79.3088: dragged up by the long right tail.
- ⑤ Mode = e^(μ−σ²) 45.4537: the most frequent failure age.
- ⑥ B10 life 30.1776 [18.953, 48.0498]: one shaft in ten has failed by this age.
- ⑦ R(100) — survival 0.2466292029: about one shaft in four survives 100 thousand cycles.

| | INTERPRETATION |
|---|---|
| 1 | ln T follows a Normal distribution — the model of MULTIPLICATIVE damage (each stage grows the flaw in proportion to its size: fatigue cracks). Mind the #1 trap: μ and σ describe the LOG of the data (log-hours, log-cycles), not the data itself. The three centers are always ordered Mode (45.4537) < Median (65.8775) < Mean (79.3088) — the right tail drags the mean up, so lognormal work reports the median with percentiles, never the mean alone. The hazard rises then FALLS at old age — adequate for fatigue-type populations, wrong for pure wear-out (that is the gamma process / Weibull territory). |

Interpretation

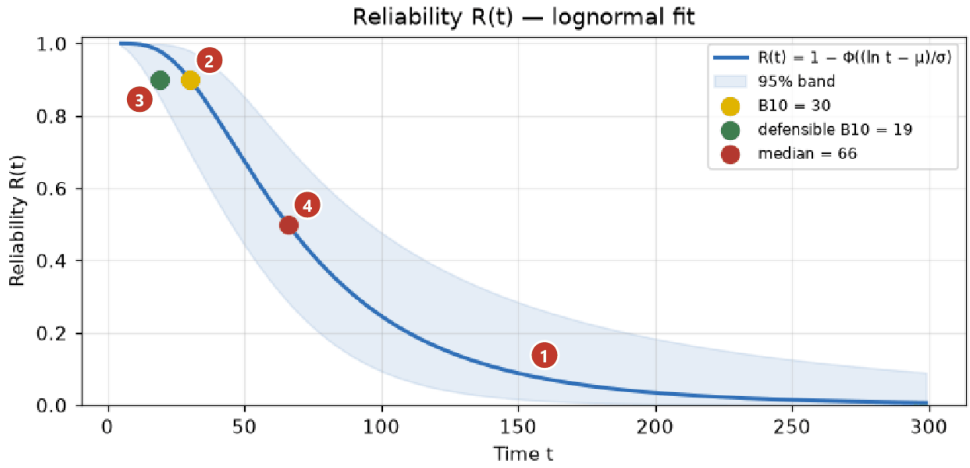
- ① ReliaEn's reading: ln T follows a Normal distribution — the model of MULTIPLICATIVE damage (each stage grows the flaw in proportion to its size: fatigue cracks). Mind the #1 trap: μ and σ describe the LOG of the data (log-hours, log-cycles), not the data itself. The three centers are always ordered Mode (45.4537) < Median (65.8775) < Mean (79.3088) — the right tail drags the mean up, so lognormal work reports the median with percentiles, never the mean alone. The hazard rises then FALLS at old age — adequate for fatigue-type populations, wrong for pure wear-out (that is the gamma process / Weibull territory).




Interactive Plot

The lognormal probability plot

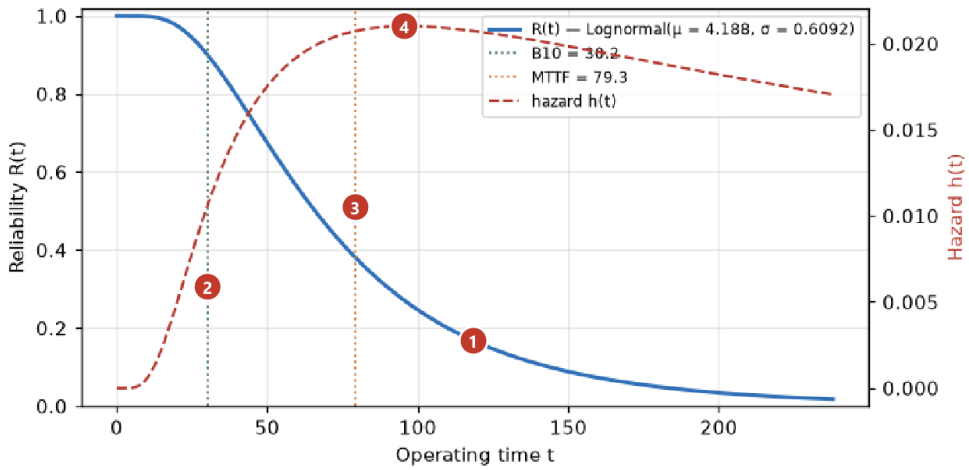
- ① failures (Benard ranks): the 12 ages on lognormal paper.
- ② MLE: μ = 4.188, σ = 0.609: the fitted line.
- ③ rank regression: σ = 0.688 (r = 0.9997): the plot's own straight-line fit, for comparison.




Interactive Plot

Reliability with its band

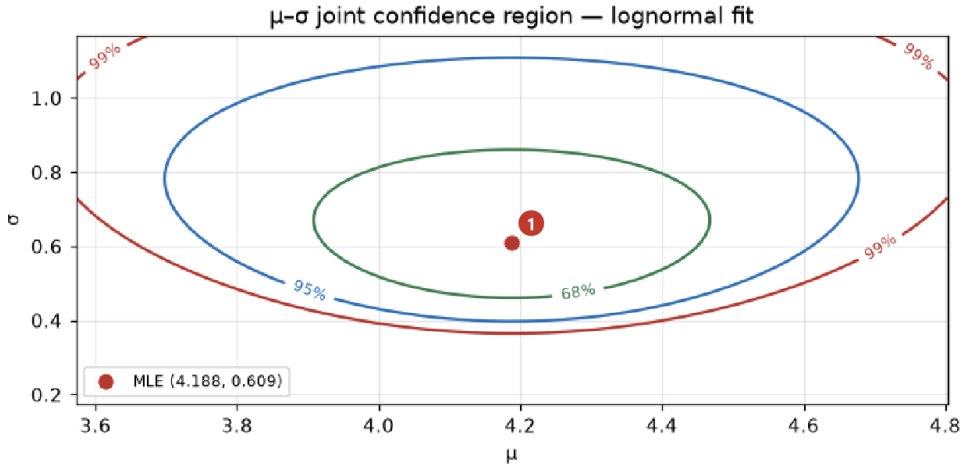
- ① R(t) = 1 − Φ((ln t − μ)/σ): the share of shafts still running at each age, inside its 95% band.
- ② B10 = 30: where R crosses 90%.
- ③ defensible B10 = 19: the lower bound of B10 - the figure the data defend.
- ④ median = 66: where R crosses 50%.




Interactive Plot

Reliability and hazard

- ① R(t) — Lognormal(μ = 4.188, σ = 0.6092): the reliability curve.
- ② B10 = 30.2.
- ③ MTTF = 79.3.
- ④ hazard h(t): rising, then falling at old age - the lognormal signature.




Interactive Plot

The joint confidence region of μ and σ

- ① MLE (4.188, 0.609): the estimate, inside its 68%, 95% and 99% regions.

**ESTIMATION METHOD COMPARISON — LOGNORMAL FIT**

1 MLE is the reported model (likelihood-highest); the rank-regression line is the probability plot's own slope/ intercept — shown for cross-checking against rank-regression-based software.

| 2 Method | 3 μ | 4 σ | 5 Median | 6 B10 | 7 r |
|---|---|---|---|---|---|
| 8 MLE (reported) | 4.1878 | 0.6092 | 65.8775 | 30.1776 | – |
| 9 Rank regression (plot line) | 4.1878 | 0.6880 | 65.8775 | 27.2793 | 0.9997 |

Estimation method comparison — lognormal fit

- ① MLE is the reported model (likelihood-highest); the rank-regression line is the probability plot's own slope/ intercept — shown for cross-checking against rank-regression-based software.
- ② to ⑦ The columns Method, μ, σ, Median, B10, r.
- ⑧ MLE (reported): μ 4.1878, σ 0.6092, median 65.8775, B10 30.1776.
- ⑨ Rank regression (plot line): μ 4.1878, σ 0.6880, median 65.8775, B10 27.2793, r 0.9997.

**PROBABILITY-PLOT FIT CHECK (PPCC, 5% LEVEL) — LOGNORMAL FIT**

1 Straightness NOT rejected (r = 0.9997 ≥ critical 0.9268).

2 r measures how straight the points lie on lognormal paper; the critical value is what true lognormal data of this sample size would rarely fall below (seeded simulation).

Probability-plot fit check (PPCC, 5% level) — lognormal fit

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- ② r measures how straight the points lie on lognormal paper; the critical value is what true lognormal data of this sample size would rarely fall below (seeded simulation).

**FORMAL TESTS ON LN T — LOGNORMAL FIT**

1 'Not rejected' is not proof — combined with a straight plot and the right physics (multiplicative damage) it is enough to use the model with confidence.

| 2 Test | 3 Statistic | 4 Criterion | 5 Verdict |
|---|---|---|---|
| 6 Shapiro–Wilk | W = 0.994 | p = 1.000 vs 0.05 | not rejected ✓ |
| 7 Anderson–Darling | $A^2$ = 0.075 | critical (5%) = 0.698 | not rejected ✓ |

Formal tests on ln t — lognormal fit

- ① 'Not rejected' is not proof — combined with a straight plot and the right physics (multiplicative damage) it is enough to use the model with confidence.
- ② to ⑤ The columns Test, Statistic, Criterion, Verdict.
- ⑥ Shapiro–Wilk: W = 0.994, p = 1.000 vs 0.05, not rejected ✓.
- ⑦ Anderson–Darling: $A^2$ = 0.075, critical (5%) = 0.698, not rejected ✓.

**COMPETITOR MODEL — LOGNORMAL VS WEIBULL (LOGNORMAL FIT)**

1 Lognormal explains this data better (by AIC).

| 2 Model | 3 Parameters | 4 ln-likelihood | 5 AIC (lower wins) |
|---|---|---|---|
| 6 Lognormal | μ = 4.188, σ = 0.609 | -61.33 | 126.7 |
| 7 Weibull | β = 1.773, η = 89 | -61.83 | 127.7 |

8 At small n these two models are often hard to separate by data alone — the final tiebreaker is the physics: multiplicative growth (fatigue, repair delays) → Lognormal; weakest-link / flexible hazard → Weibull.

Competitor model — Lognormal vs Weibull (lognormal fit)

- ① Lognormal explains this data better (by AIC).
- ② to ⑤ The columns Model, Parameters, ln-likelihood, AIC (lower wins).
- ⑥ Lognormal: μ = 4.188, σ = 0.609, -61.33, AIC 126.7.
- ⑦ Weibull: β = 1.773, η = 89, -61.83, AIC 127.7.
- ⑧ At small n these two models are often hard to separate by data alone — the final tiebreaker is the physics: multiplicative growth (fatigue, repair delays) → Lognormal; weakest-link / flexible hazard → Weibull.

**B-LIFE TABLE — LOGNORMAL FIT**

1 Bα = exp(μ + z_α σ): the age by which α % of the population has failed. Left-tail protection — pick the α your risk criterion allows.

| 2 B-life | 3 Time | 4 95% bounds |
|---|---|---|
| 5 B1 | 15.9686 | [8.22441, 31.005] |
| 6 B5 | 24.1862 | [14.2549, 41.0366] |
| 7 B10 | 30.1776 | [18.953, 48.0498] |
| 8 B50 | 65.8775 | [46.6712, 92.9878] |

B-life table — lognormal fit

- ① Bα = exp(μ + z_α σ): the age by which α % of the population has failed. Left-tail protection — pick the α your risk criterion allows.
- ② to ④ The columns B-life, Time, 95% bounds.
- ⑤ B1 15.9686 [8.22441, 31.005].
- ⑥ B5 24.1862 [14.2549, 41.0366].
- ⑦ B10 30.1776 [18.953, 48.0498].
- ⑧ B50 65.8775 [46.6712, 92.9878].

**VALUE OF MORE DATA — DEFENSIBLE B10 VS TESTS RUN (LOGNORMAL FIT)**

1. More tests narrow the bound: from 18.953 today toward the point estimate 30.1776 — this table prices the next test campaign before it is bought.

| Complete tests | Defensible B10 (95% lower bound) |
|---|---|
| today n = 12 | 18.953 |
| n = 25 | 21.864 |
| n = 30 | 22.4866 |
| n = 35 | 22.9827 |
| n = 40 | 23.3905 |
| n = 50 | 24.0282 |

8. Projection holds today's estimates fixed and shrinks only the Fisher uncertainty (se ∝ 1/√n) — it prices INFORMATION, not fate; new data may also move the estimates.

Value of more data — defensible B10 vs tests run (lognormal fit)

- ① More tests narrow the bound: from 18.953 today toward the point estimate 30.1776 — this table prices the next test campaign before it is bought.
- ② today n = 12: 18.953.
- ③ n = 25: 21.864.
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- ⑥ n = 40: 23.3905.
- ⑦ n = 50: 24.0282.
- ⑧ Projection holds today's estimates fixed and shrinks only the Fisher uncertainty (se ∝ 1/√n) — it prices INFORMATION, not fate; new data may also move the estimates.

**THE Σ LEVER — WHAT REDUCING SCATTER BUYS (B10, LOGNORMAL FIT)**

1. A longer replacement interval is EARNED by narrowing the spread (surface finish, heat treatment, process control) — not by stretching the number. This table prices the improvement.

| Fleet quality | B10 |
|---|---|
| today (σ = 0.609) | 30.1776 |
| scatter −10% (σ = 0.548) | 32.6279 |
| scatter −20% (σ = 0.487) | 35.2773 |
| scatter −30% (σ = 0.426) | 38.1417 |

6. Every B-level below the median rises as σ falls (median itself is unchanged: e^μ).

The σ lever — what reducing scatter buys (B10, lognormal fit)

- ① A longer replacement interval is EARNED by narrowing the spread (surface finish, heat treatment, process control) — not by stretching the number. This table prices the improvement.
- ② today (σ = 0.609): B10 30.1776.
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- ④ scatter −20% (σ = 0.487): 35.2773.
- ⑤ scatter −30% (σ = 0.426): 38.1417.
- ⑥ Every B-level below the median rises as σ falls (median itself is unchanged: e^μ).

**SERVICE INTERVALS — OPERATING TIME TO STAY AT OR ABOVE TARGET RELIABILITY**

1 Model: Lognormal(μ = 4.188, σ = 0.6092) — service before the unit accumulates this much operating time to keep reliability at or above the target.

| 2 Target R | 3 Operating time t |
|---|---|
| 95% | 24.2 |
| 4 90% | 30.2 |
| 85% | 35.0 |
| 80% | 39.5 |
| 75% | 43.7 |
| 70% | 47.9 |
| 65% | 52.1 |
| 60% | 56.5 |
| 55% | 61.0 |
| 50% | 65.9 |
| 45% | 71.1 |
| 40% | 76.9 |
| 35% | 83.3 |
| 30% | 90.7 |
| 25% | 99.4 |
| 20% | 110.0 |
| 15% | 123.9 |
| 10% | 143.8 |
| 5% | 179.4 |

SERVICE INTERVALS — operating time to stay at or above target reliability

- ① Model: Lognormal(μ = 4.188, σ = 0.6092) — service before the unit accumulates this much operating time to keep reliability at or above the target.
- ② Target R: the reliability to keep, from 95% down to 5%.
- ③ Operating time t: the longest run that keeps it.
- ④ The 90% row: 30.2 thousand cycles - B10.

PERCENTILE TABLE — B-LIVES WITH BOUNDS

1 B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.

| 2 Percentile | 3 B-life | 4 95% bounds |
|---|---|---|
| 1% | 15.97 | [8.224, 31] |
| 5% | 24.19 | [14.25, 41.04] |
| 5 10% | 30.18 | [18.95, 48.05] |
| 25% | 43.68 | [29.82, 63.99] |
| 50% | 65.88 | [46.67, 92.99] |
| 75% | 99.35 | [67.82, 145.6] |
| 90% | 143.8 | [90.32, 229] |
| 95% | 179.4 | [105.8, 304.4] |
| 99% | 271.8 | [140, 527.7] |

Percentile table — B-lives with bounds

- ① B_p = the age by which p% of units fail. Bounds by each family's own interval machinery; a — cell means no bound machinery exists for that family — the point estimate stands alone.
- ② to ④ The columns Percentile, B-life, 95% bounds.
- ⑤ The 10% row: 30.18 [18.95, 48.05] - the B10 of the results card, to four figures.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

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# Part 5 — How to interpret the results

**Read μ and σ on the log scale.** $\hat{\mu}$ = 4.1878 and $\hat{\sigma}$ = 0.609183 describe ln t, not t. The life figures are their translation: half the shafts fail by 65.8775 thousand cycles (the median), one in ten by 30.1776 (B10), and the mean 79.3088 sits above the median because of the long right tail - report the median with its percentiles, never the mean alone.

**What the data defend.** B10's lower bound is 18.953: twelve shafts defend only that much. About one shaft in four survives 100 thousand cycles (R(100) = 0.2466292029). The plot, the PPCC and both formal tests do not reject the lognormal model, and it edges the Weibull by AIC (126.7 against 127.7) - a gap too small to prove anything on its own; the physics, fatigue cracks that grow, points to the lognormal.

**What lengthens B10.** A narrower scatter is the fastest route: 30% less scatter lifts B10 from 30.1776 to 38.1417, through surface finish, heat treatment or process control. More tests raise the defensible B10 only slowly - from 18.953 today to 24.0282 at 50 shafts.