Goodness of fit testing — is the failure rate constant?

Menu: Life Data ▸ Goodness of Fit Testing…

Part 1 — What the function does

What it calculates. The function tests whether a set of failure ages is consistent with a **constant failure rate**: the exponential model, in which a used unit is as likely to fail in the next hour as a new one. It sorts the failure ages into equal-width age intervals, counts the failures observed in each interval, and compares them with the failures a constant rate would put there. The comparison is the chi-square statistic χ^2_0. ReliaEn prints:

What ReliaEn prints	What it tells you
λ, the constant failure rate	The rate the test assumes, in failures per unit of operating time
The **Interval comparison** table	Observed failures O_i against expected failures E_i in each age interval, and each interval's share of χ^2_0
χ^2_0 and **χ^2crit**	The test statistic, and the largest value a constant rate still allows at your significance level
The **Verdict**	CONSTANT rate confirmed or constant rate REJECTED, with the next analysis to run

From which data. The ages at failure of a group of like units, all of which failed, in one worksheet column, one row per unit. At least 5 ages; every age above 0, in one consistent unit of operating time (hours, days, cycles).

When an engineer uses it. Before planning with a constant failure rate. A constant rate is the basis of the exponential analyses (Life Data ▸ Estimate Failure Function), of spare-parts sizing from an MTBF, and of the rule that preventive replacement cannot pay. The test decides whether that basis holds or whether the rate changes with age, in which case the next step is the Weibull model.

When not to use it.

Situation	Use instead
Some units were still running when you stopped observing	Life Data ▸ Time Dependence ▸ Weibull (2P) distribution… (it uses the survivors)
One repairable system fails again and again	Life Data ▸ Estimate Failure Function ▸ Repairable unit data…
You already know the rate is constant and want its bounds	Life Data ▸ Estimate Failure Function ▸ Complete data…

Part 2 — The example's input data

Setting. A water utility records the ages at failure of twenty identical booster-pump seals, in operating days. The maintenance planner wants to know whether the seals fail at a constant rate before sizing the spare-seal stock from an MTBF.

Input	Value
Failure ages, operating days (20)	23, 48, 61, 87, 102, 131, 149, 175, 212, 234, 268, 291, 322, 355, 389, 421, 456, 484, 521, 568

File in the worked-example zip	2-Goodness-of-Fit/Example-1.csv — one column, heading Failure time, 20 rows
Interval width	100 operating days
Other settings	Significance % 5, λ estimate Interval estimate, λ confidence % 60 — the values the form opens with

	① A	B
1	23	
2	48	
3	61	
4	87	
5	102	
6	131	
7	149	
8	175	
9	212	
10	234	
11	268	
12	291	
13	322	
14	355	
15	389	
16	421	
17	456	
18	484	
19	521	
20	568	
21		

The example data in the worksheet

- ① Column A: the 20 failure ages, one per row, in operating days.

Part 3 — How to use it, step by step

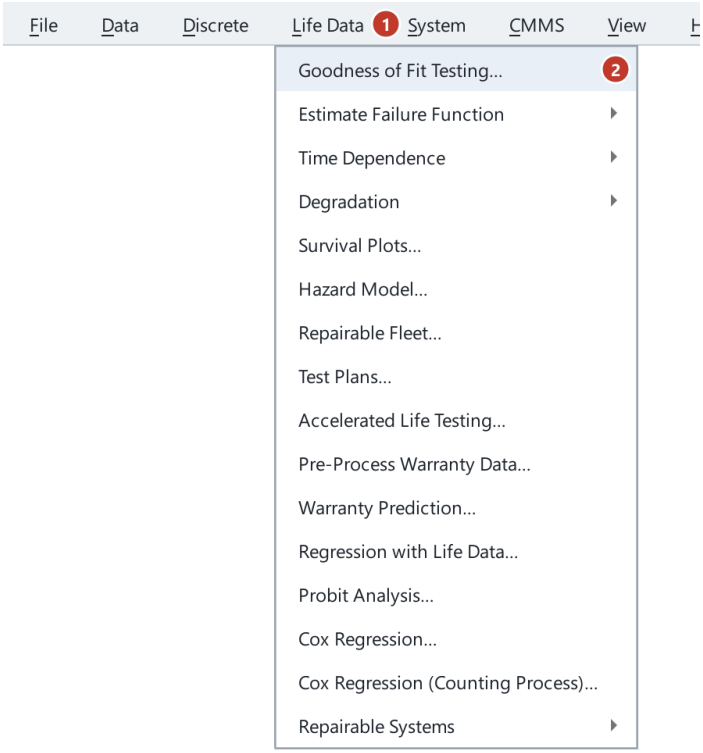
1. **Put the failure ages in column A.** Copy the 20 values of Example-1.csv that stand under its heading Failure time into column A of the worksheet, starting at row 1.

	① A	B
1	23	
2	48	
3	61	
4	87	
5	102	
6	131	
7	149	
8	175	
9	212	
10	234	
11	268	
12	291	
13	322	
14	355	
15	389	
16	421	
17	456	
18	484	
19	521	
20	568	
21		

Step 1 — the failure ages in column A

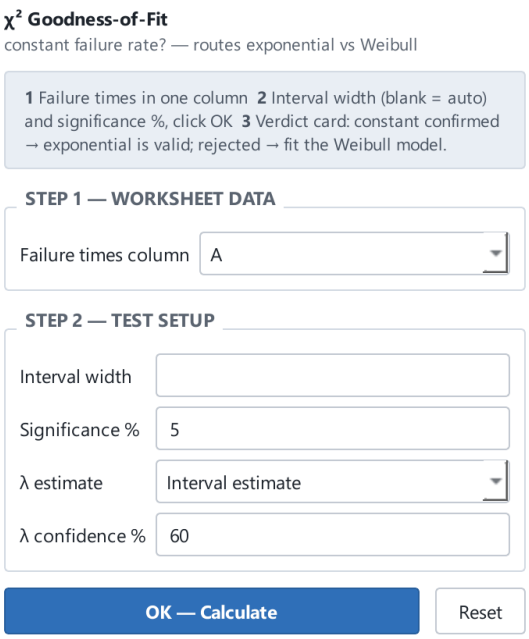
- ① Column A: the 20 failure ages, 23 to 568.

2. **Choose Life Data ▸ Goodness of Fit Testing...** in the menu bar. The form χ² Goodness-of-Fit opens in the left panel.



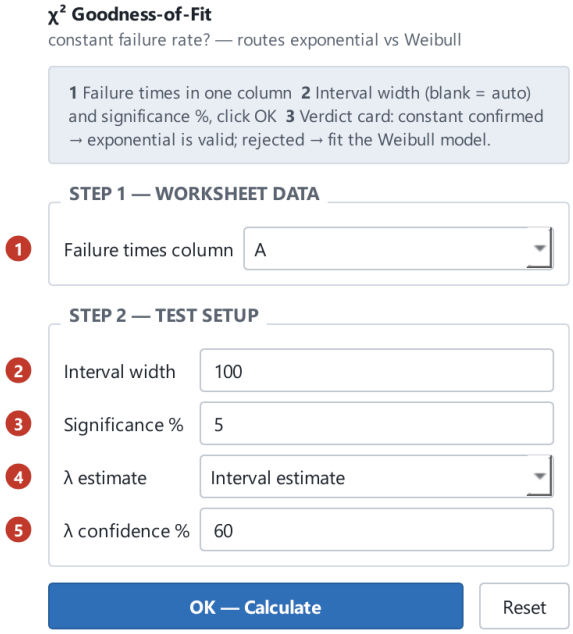
Step 2 — the menu path

- ① Life Data: the menu.
- ② Goodness of Fit Testing...: the entry to click.



Step 2 — the form as it opens: Interval width is blank, the other boxes hold their defaults

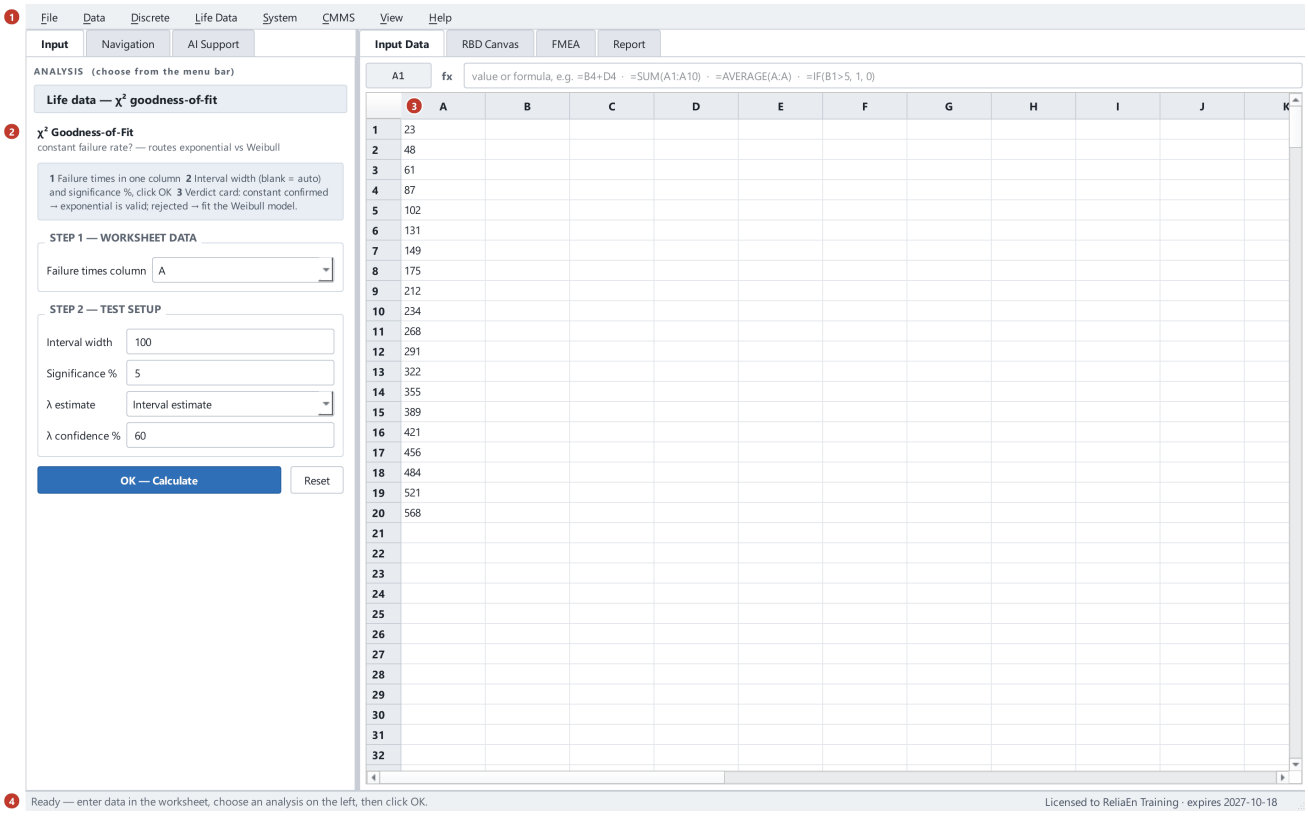
Steps 3 to 7 set the form's five boxes, marked ① to ⑤ in the picture:



Steps 3 to 7 — the form set for this example

3. ① **In Failure times column, choose A.** This is the column that holds the failure ages.
4. ② **Type 100 in Interval width.** The ages are sorted into the intervals 0 – 100, 100 – 200 and so on, in operating days; the last interval runs to infinity. Left blank, the width would be the largest age divided by 6.
5. ③ **Check that Significance % reads 5.** The test rejects a constant rate only when the data would occur by chance less than 5 times in 100 under a constant rate.
6. ④ **In λ estimate, choose Interval estimate.** The rate is then the upper confidence bound χ^2(confidence, 2r) / 2T, which is slightly conservative; Point estimate would use r / T.
7. ⑤ **Check that λ confidence % reads 60.** The confidence of the interval estimate of the rate; 60% is the course convention.

8. **Click OK — Calculate** at the foot of the form (② in the picture below). ReliaEn runs the test and shows the report.



Step 8 — the window just before OK

- ① The menu bar: File, Data, Discrete, Life Data, System, CMMS, View, Help.
- ② The form, set as in steps 3 to 7; OK — Calculate at its foot.
- ③ Column A: the 20 failure ages.
- ④ The status bar: Ready — enter data in the worksheet, choose an analysis on the left, then click OK. and Licensed to ReliaEn Training · expires 2027-10-18.

Part 4 — The report, card by card

Life Data — χ² Goodness-of-Fit (constant rate?)

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$E_i = N\cdot[e^{-\lambda a} - e^{-\lambda b}]$; $\chi^2_o = \Sigma(E_i-O_i)^2/E_i$; dof = intervals − 2

The report's head

- The title Life Data — χ² Goodness-of-Fit (constant rate?), the date and time of the run, and the method in one line: $E_i = N\cdot[e-\lambda a - e-\lambda b]$; $\chi^2_o = \Sigma(E_i-O_i)^2/E_i$; dof = intervals − 2.

	INPUT DATA	
1	Failure data	r = 20 failures · T = ΣTTF = 5297
2	Interval width w	100
3	λ estimate	0.00392885 (Interval estimate — χ²(60%, 2r) / 2T)
4	Intervals (equal width, last → ∞)	6
5	Significance level	5%

Input data

- ① Failure data r = 20 failures · T = ΣTTF = 5297: ReliaEn read 20 failure ages, and their total is 5,297 operating days. Check r first: a blank cell is skipped, and r would then be lower than the rows you entered.
- ② Interval width w 100: the width typed in step 4.
- ③ λ estimate 0.00392885, with the rule used, (Interval estimate — χ^2(60%, 2r) / 2T): the constant rate the test assumes, about 3.9 failures per 1,000 operating days.
- ④ Intervals (equal width, last → ∞) 6: 0 – 100 up to 400 – 500, and 500 – ∞.
- ⑤ Significance level 5%.

	RESULTS	
1	$\chi^2_0 = \Sigma(E_i-O_i)^2/E_i$	4.10774
2	χ^2crit (95%, dof = 4)	9.48773
3	Verdict	CONSTANT rate confirmed

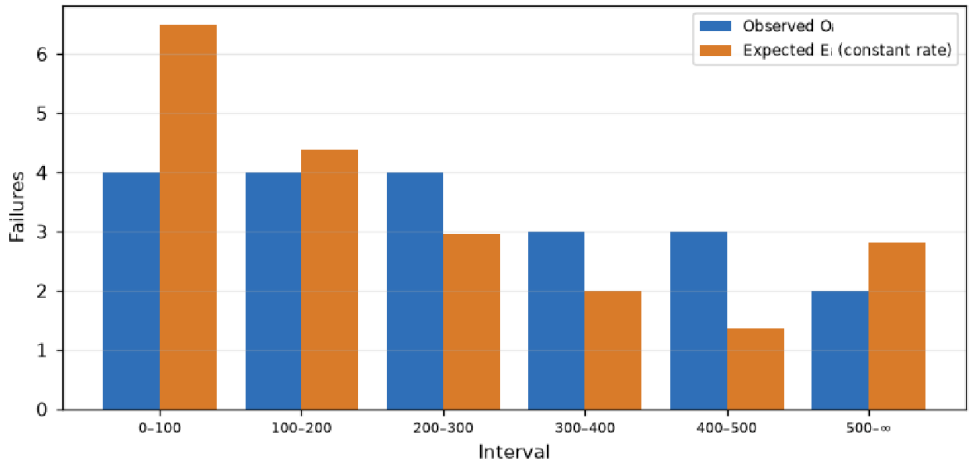
Results

- ① $\chi^2_0 = \Sigma(E_i-O_i)^2/E_i$ 4.10774: how far the observed counts are from the constant-rate counts, summed over the 6 intervals.
- ② χ^2crit (95%, dof = 4) 9.48773: the largest χ^2_0 that a constant rate allows at 5% significance with 4 degrees of freedom (6 intervals − 2).
- ③ Verdict CONSTANT rate confirmed: 4.10774 ≤ 9.48773.

	INTERPRETATION
1	χ^2_0 = 4.10774 ≤ χ^2crit = 9.48773 — the constant-failure-rate hypothesis is NOT rejected at the 5% significance level. The exponential model (λ = 0.00392885) is valid for this data: use the exponential life-data analyses. Preventive replacement cannot pay under a constant rate (D23).

Interpretation

- ① ReliaEn's sentence: χ^2_0 = 4.10774 ≤ χ^2crit = 9.48773 — the constant-failure-rate hypothesis is NOT rejected at the 5% significance level. The exponential model (λ = 0.00392885) is valid for this data: use the exponential life-data analyses. Preventive replacement cannot pay under a constant rate (D23). The comparison, the verdict, the next analysis, and the maintenance consequence, in that order.



Interactive Plot

Observed against expected failures per interval

- The blue bars, Observed O_i: the seals that failed in each interval — 4, 4, 4, 3, 3 and 2.
- The orange bars, Expected E_i (constant rate): the failures a constant rate puts there. They fall from 6.498 in 0 – 100 to 1.350 in 400 – 500, and rise again in 500 – ∞ (2.805), because the last interval is open and collects every later failure.
- Where blue and orange differ most: 0 – 100 (4 observed against 6.498 expected) and 400 – 500 (3 against 1.350). Under the chart is the Interactive Plot link.

INTERVAL COMPARISON

1 Σ row: $\Sigma p_i = 1$ and $\Sigma E_i = r$ by construction (telescoping identity) — the whole-table integrity check.

2 Interval	3 Observed O_i	4 P_i	5 Expected E_i	6 $(E_i-O_i)^2/E_i$
0 – 100	4	0.3249	6.498	0.9602
100 – 200	4	0.2193	4.387	0.0341
200 – 300	4	0.1481	2.962	0.3641
300 – 400	3	0.1000	1.999	0.5008
400 – 500	3	0.0675	1.350	2.0176
500 – ∞	2	0.1402	2.805	0.2309
Σ	20	1.0000	20.000	4.1077

Interval comparison

- ① The note above the table, Σ row: $\Sigma p_i = 1$ and $\Sigma E_i = r$ by construction (telescoping identity) — the whole-table integrity check.: if the Σ row did not read 1.0000 and 20.000, the table would be wrong.
- ② Interval: the six age intervals, 0 – 100 to 500 – ∞ operating days.
- ③ Observed O_i: the seals that failed in the interval — 4, 4, 4, 3, 3, 2; the Σ row is 20.
- ④ P_i: the probability that a seal fails in the interval under the constant rate. The column sums to 1.
- ⑤ Expected E_i = 20 × P_i: the failures a constant rate puts in the interval. The column sums to 20, the number of failures.
- ⑥ $(E_i-O_i)^2/E_i$: each interval's share of χ^2_0; the Σ row is χ^2_0 itself, 4.1077.

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The report's foot

- ① The licence line Licensed to ReliaEn Training · RL-2026-0017: the licence the report was made under.

Part 5 — How to interpret the results

The decision. χ^2_0 = 4.10774 is well below χ^2crit = 9.48773, so the test does not reject a constant failure rate at 5% significance. For these twenty seals the planner may use the exponential model. The rate the test used, λ estimate 0.00392885 failures per operating day, is the 60% upper bound and so errs on the safe side; the exponential analyses under Life Data ▸ Estimate Failure Function give the rate with its full bounds.

What follows for maintenance. Under a constant rate a used seal is as good as a new one, so replacing seals at a fixed age cannot reduce failures; ReliaEn's interpretation says so. Plan the seals as run-to-failure parts and size the spare stock from the rate (System ▸ Spare Parts Optimization…).

Where the data strain the model. The largest contribution to χ^2_0, 2.0176, comes from the interval 400 – 500 days, where three seals failed against 1.350 expected. One such interval does not reject the model, but it is the place to watch: if later failures keep gathering at high ages, re-run the test with the new ages; a rejection would point to wear-out and to the Weibull model (Life Data ▸ Time Dependence ▸ Weibull (2P) distribution…).

What a pass does not prove. The verdict means the data cannot reject a constant rate; it does not prove the rate is constant. With 20 failures and 6 intervals the expected counts are small (1.350 to 6.498), so a slow wear-out could still pass. Read the verdict together with what you know about how the seals fail.